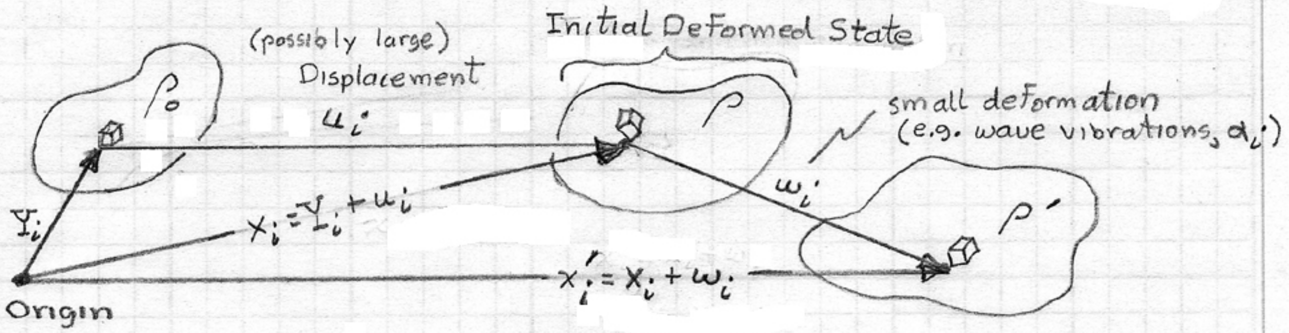
Derivation of Acousto-Elastic Effect (Lagrangian)



In any deformed state the symmetric Cauchy stress tensor is given by:

$$t_{kl} = \frac{\rho}{\rho_o}\frac{\partial \Sigma}{\partial E_{mn}}\frac{\partial x_k}{\partial Y_m}\frac{\partial x_l}{\partial Y_n} \quad (1)$$

where $\Sigma = \Sigma(E_{mn})$ is the strain energy density, N-m/m^3, which is a function only of strain, E_{mn}.

OR

u = strain energy/mass [=] N-m/kg

$\tilde{u}$ = strain energy density [=] N-m/m^3

$\frac{N\text{-}m}{kg}\frac{kg}{m^3}$ [=] $u \cdot \rho_o = \tilde{u}$ [=] $\frac{N\text{-}m}{m^3} = \frac{N}{m^2}$

$$\sigma_{ij} = \rho\frac{\partial u}{\partial l_{ij}} \quad \text{Ref.1 eqn. (5.88)}$$

$t_{ij} = \sigma_{ij}$, $E_{ij} = l_{ij}$, $\tilde{u} = \Sigma$

$$t_{ij} = \frac{\rho}{\rho_o}\frac{\partial \tilde{u}}{\partial E_{ij}} = \frac{\rho}{\rho_o}\frac{\partial \Sigma}{\partial E_{mn}}\frac{\partial x_i}{\partial Y_m}\frac{\partial x_j}{\partial Y_n} \quad (1)$$

The Green-St. Venant or Lagrangian strain-displacement relationship

$$E_{kl} = \frac{1}{2}\left\{\frac{\partial u_k}{\partial Y_l} + \frac{\partial u_l}{\partial Y_k} + \frac{\partial u_m}{\partial Y_k}\cdot\frac{\partial u_m}{\partial Y_l}\right\} = l_{kl} \quad (2)$$

Let the unprimed quantity, x_i, described the initial deformed state. Similarly primed quantities, ρ' and x_i', refer to the final state.

$$\text{Let} \quad \delta E_{kl} = E'_{kl} - E_{kl} \quad (3)$$

$$\text{where} \quad E'_{kl} = \frac{1}{2}\left\{\frac{\partial}{\partial Y_l}(u_k + w_k) + \frac{\partial}{\partial Y_k}(u_l + w_l) + \frac{\partial}{\partial Y_k}(u_m + w_m)\cdot\frac{\partial}{\partial Y_l}(u_m + w_m)\right\} \quad (4)$$

Expand and keep only first order terms $\partial w_m/\partial Y_n$ and recall $x_m = Y_m + u_m$

1. Frederick and Chang, "Continuum Mechanics", Scientific Publishers, Boston, 1972.

* Rewritten with additions and comments by Dr. Ron Kriz for ESM 5344 Department of Engineering Science and Mechanics, Virginia Tech, 2005

and $\delta_{Km} \equiv \partial Y_m/\partial Y_K$ subst. for $Y_m = x_m - u_m$, $\frac{\partial x_m}{\partial Y_K} = \delta_{Km} + \frac{\partial u_m}{\partial Y_K}$, when substituted into (3) yields

$$\delta E_{Kl} = \frac{1}{2}\left\{ \frac{\partial w_K}{\partial Y_l} + \frac{\partial w_l}{\partial Y_K} + \frac{\partial w_m}{\partial Y_K}\cdot\frac{\partial u_m}{\partial Y_l} + \frac{\partial w_m}{\partial Y_l}\cdot\frac{\partial u_m}{\partial Y_K} \right\} \quad (5a)$$

$$= \frac{1}{2}\left\{ \frac{\partial w_m}{\partial Y_l}\left(\delta_{km} + \frac{\partial u_m}{\partial Y_K}\right) + \frac{\partial w_m}{\partial Y_K}\left(\delta_{lm} + \frac{\partial u_m}{\partial Y_l}\right) \right\} \quad (5b)$$

$$= \frac{1}{2}\left\{ \frac{\partial w_m}{\partial Y_l}\frac{\partial x_m}{\partial Y_K} + \frac{\partial w_m}{\partial Y_K}\frac{\partial x_m}{\partial Y_l} \right\} \quad (5c)$$

$$\delta E_{Kl} = \frac{1}{2}\left\{ \frac{\partial w_K}{\partial Y_l} + \frac{\partial w_l}{\partial Y_K} \right\}$$ since $\frac{\partial x_m}{\partial Y_K} = a_{mK}$ and $\frac{\partial x_m}{\partial Y_l} = a_{ml}$

The Cauchy stress in the final (primed) state:

$$t'_{Kl} = \frac{\rho'}{\rho_o}\left(\frac{\partial \Sigma}{\partial E'_{mn}}\right)\frac{\partial x'_K}{\partial Y_m}\frac{\partial x'_l}{\partial Y_n} \quad (6)$$

where $x'_K = x_K + w_K$ and expand the terms in parantheses, using a Taylor series expansion if $\delta E'_{mn}$ is small

$$\frac{\partial \Sigma(E'_{mn})}{\partial E'_{mn}} = \frac{\partial \Sigma}{\partial E_{mn}} + \frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\delta E_{pq}$$

then (6) can be rewritten to first order in $\partial \underset{\sim}{w}/\partial \underset{\sim}{Y}$

$$t'_{Kl} = \frac{\rho'}{\rho_o}\left\{\frac{\partial \Sigma}{\partial E_{mn}} + \frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\delta E_{pq}\right\}\left\{\frac{\partial x_K}{\partial Y_m} + \frac{\partial w_K}{\partial Y_m}\right\}\left\{\frac{\partial x_l}{\partial Y_n} + \frac{\partial w_l}{\partial Y_n}\right\} \quad (7)$$

using $\rho'/\rho_o = \rho/\rho_o + (\rho' - \rho)/\rho \cdot \rho/\rho_o$ equation (7) can be rewritten

$$t'_{Kl} = \frac{\rho}{\rho_o}\left\{\quad " \quad\right\}\left\{\quad " \quad\right\}\left\{\quad " \quad\right\} \text{ 1rst term}$$

$$+ \frac{\rho' - \rho}{\rho}\cdot\frac{\rho}{\rho_o}\left\{\quad " \quad\right\}\left\{\quad " \quad\right\}\left\{\quad " \quad\right\} \text{ 2nd term} \quad (8)$$

where $\frac{\rho' - \rho}{\rho} = \frac{\delta \rho}{\rho} = -\frac{\delta V}{V} = -\frac{\partial w_\alpha}{\partial x_\alpha}$ (9), is a dilatation to first order

Equation (8) can be rewritten keeping only first order terms $\frac{\partial \underset{\sim}{w}}{\partial \underset{\sim}{x}}$

The second term of (8) reduces to

$$-\frac{\partial w_\alpha}{\partial x_\alpha}\frac{\rho}{\rho_0}\frac{\partial \Sigma}{\partial E_{mn}}\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n} = -\frac{\partial w_\alpha}{\partial x_\alpha} t_{K\ell} \qquad (10)$$

Using (5c), which is first order in $\partial \underset{\sim}{w}/\partial \underset{\sim}{Y}$, the first term in equation (8) reduces, keeping only first order terms $\partial \underset{\sim}{w}/\partial \underset{\sim}{Y}$,

$$+\frac{\rho}{\rho_0}\frac{\partial \Sigma}{\partial E_{mn}}\left\{\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n} + \frac{\partial x_K}{\partial Y_m}\frac{\partial w_\ell}{\partial Y_n} + \frac{\partial x_\ell}{\partial Y_n}\frac{\partial w_K}{\partial Y_m}\right\}$$

$$+\frac{\rho}{\rho_0}\frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n}\frac{1}{2}\left\{\frac{\partial w_\beta}{\partial Y_p}\frac{\partial x_\beta}{\partial Y_q} + \frac{\partial w_\beta}{\partial Y_q}\frac{\partial x_\beta}{\partial Y_p}\right\} \qquad (11)$$

Equation (11) can be further reduced using symmetry of the strain tensor together with equation (1)

$$t_{K\ell} + \frac{\rho}{\rho_0}\frac{\partial \Sigma}{\partial E_{mn}}\left\{\frac{\partial x_K}{\partial Y_m}\frac{\partial w_\ell}{\partial Y_n} + \frac{\partial x_\ell}{\partial Y_n}\frac{\partial w_K}{\partial Y_m}\right\}$$

$$+\frac{\rho}{\rho_0}\frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n}\frac{\partial w_\beta}{\partial Y_p}\frac{\partial x_\beta}{\partial Y_q} \qquad (12)$$

Using $\frac{\partial w_\ell}{\partial Y_n} = \frac{\partial w_\ell}{\partial x_\beta}\frac{\partial x_\beta}{\partial Y_n}$ (13) equations (10) and (12) combine

$$t'_{K\ell} = t_{K\ell} - \frac{\partial w_\alpha}{\partial x_\alpha} t_{K\ell} + \frac{\rho}{\rho_0}\frac{\partial \Sigma}{\partial E_{mn}}\left\{\frac{\partial w_\ell}{\partial x_\beta}\frac{\partial x_\beta}{\partial Y_n}\frac{\partial x_K}{\partial Y_m} + \frac{\partial w_K}{\partial x_\beta}\frac{\partial x_\beta}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n}\right\}$$

$$+\frac{\rho}{\rho_0}\frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n}\frac{\partial x_\beta}{\partial Y_q}\frac{\partial w_\beta}{\partial x_\sigma}\frac{\partial x_\sigma}{\partial Y_p} \qquad (14)$$

and using (1),

$$t'_{K\ell} - t_{K\ell} = -\frac{\partial w_\alpha}{\partial x_\alpha} t_{K\ell} + \frac{\partial w_\ell}{\partial x_\beta} t_{\beta K} + \frac{\partial w_K}{\partial x_\beta} t_{\beta \ell}$$

$$+\frac{\rho}{\rho_0}\frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n}\frac{\partial x_\beta}{\partial Y_q}\frac{\partial x_\sigma}{\partial Y_p}\left(\frac{\partial w_\beta}{\partial x_\sigma}\right) \qquad (15)$$

Define $S_{K\ell\beta\sigma} = \frac{\rho}{\rho_0}\frac{\partial^2 \Sigma}{\partial E_{mn}\partial E_{pq}}\frac{\partial x_K}{\partial Y_m}\frac{\partial x_\ell}{\partial Y_n}\frac{\partial x_\beta}{\partial Y_q}\frac{\partial x_\sigma}{\partial Y_p}$ (16)

where $S_{k\ell\beta\sigma} = S_{\ell k\beta\sigma} = S_{k\ell\sigma\beta} = S_{\sigma\beta k\ell}$ (17)

Equation (15) can be rewritten

$$t'_{k\ell} - t_{k\ell} = -\frac{\partial w_\alpha}{\partial x_\alpha} t_{k\ell} + \frac{\partial w_\ell}{\partial x_\beta} t_{\beta k} + \frac{\partial w_k}{\partial x_\beta} t_{\beta \ell} + S_{k\ell\beta\sigma} \frac{\partial w_\beta}{\partial x_\sigma} \quad (18)$$

If the initial large deformation is static, then equilibrium equation for the initial deformed state is

$$\partial t_{k\ell} / \partial x_k = 0 \quad (19)$$

The dynamic equilibrium (equations of motion) for the final deformed state, where $\ddot{u}_\ell = 0$, are

$$\partial t'_{k\ell} / \partial x'_k = \rho' \ddot{w}_\ell \quad (20)$$

Note that $\rho' = (\rho' - \rho) + \rho$ and recall $\rho' - \rho = \delta\rho$ is first order in $\partial w_\alpha / \partial x_\alpha$ (dilatation), therefore (20) reduces to

$$\partial t'_{k\ell} / \partial x'_k = \rho \ddot{w}_\ell \quad (21)$$

Recall $x'_m = x_m + w_m$ (22)

so that $\partial / \partial x_k = \dfrac{\partial x'_s}{\partial x_k} \dfrac{\partial}{\partial x'_s} = \left(\delta_{ks} + \dfrac{\partial w_s}{\partial x_k} \right) \dfrac{\partial}{\partial x'_s}$ (23)

multiplying (23) by $\delta_{k\mu} - \partial w_k / \partial x_\mu$ yields

$$\left(\delta_{k\mu} - \frac{\partial w_k}{\partial x_\mu}\right) \frac{\partial}{\partial x_k} = \left(\delta_{k\mu} - \frac{\partial w_k}{\partial x_\mu}\right)\left(\delta_{ks} + \frac{\partial w_s}{\partial x_k}\right) \frac{\partial}{\partial x'_s}$$

$$\text{"} \quad = \left(\delta_{s\mu} - \frac{\partial w_k}{\partial x_\mu} \frac{\partial w_s}{\partial x_k}\right) \frac{\partial}{\partial x'_s} \quad (24)$$

Keeping only first order terms in (24)

$$\left(\delta_{k\mu} - \frac{\partial w_k}{\partial x_\mu}\right) \frac{\partial}{\partial x_k} = \frac{\partial}{\partial x'_\mu} \quad (25)$$

Changing indices

$$\frac{\partial}{\partial x'_k} = \left(\delta_{sk} - \frac{\partial w_s}{\partial x_k}\right) \frac{\partial}{\partial x_s} \quad (26)$$

which is used in (20)

Substituting (18) into (21) and using (26)

$$\left(\delta_{sK} - \frac{\partial w_s}{\partial x_K}\right)\frac{\partial}{\partial x_s}\left[t_{Kl} - \frac{\partial w_\alpha}{\partial x_\alpha} t_{Kl} + \frac{\partial w_l}{\partial x_\beta} t_{\beta K} + \frac{\partial w_K}{\partial x_\beta} t_{\beta l} + S_{Kl\beta\sigma}\frac{\partial w_\beta}{\partial x_\sigma}\right] = \rho \ddot{w}_l \qquad (27)$$

Keeping only first order terms in $\partial \underset{\sim}{w} / \partial \underset{\sim}{x}$ (27) reduces to

$$\frac{\partial t_{Kl}}{\partial x_K} - \frac{\partial w_s}{\partial x_K}\frac{\partial t_{Kl}}{\partial x_s} - \frac{\partial}{\partial x_K}\left\{\frac{\partial w_\alpha}{\partial x_\alpha} t_{Kl} - \frac{\partial w_l}{\partial x_\beta} t_{\beta K} - \frac{\partial w_K}{\partial x_\beta} t_{\beta l}\right\} + \frac{\partial}{\partial x_K}\left(S_{Kl\beta\sigma}\frac{\partial w_\beta}{\partial x_\sigma}\right) = \rho \ddot{w}_l \qquad (28)$$

The first term on the left side of (28) vanishes due to (19)

$$-\frac{\partial w_s}{\partial x_K}\frac{\partial t_{Kl}}{\partial x_s} - \frac{\partial^2 w_\alpha}{\partial x_K \partial x_\alpha} t_{Kl} - \frac{\partial w_\alpha}{\partial x_\alpha}\overbrace{\frac{\partial t_{Kl}}{\partial x_K}}^{0} + \frac{\partial^2 w_l}{\partial x_K \partial x_\beta} t_{\beta K} + \frac{\partial w_l}{\partial x_\beta}\overbrace{\frac{\partial t_{\beta K}}{\partial x_K}}^{0}$$
$$+ \frac{\partial w_K}{\partial x_\beta}\frac{\partial t_{\beta l}}{\partial x_K} + \frac{\partial^2 w_K}{\partial x_K \partial x_\beta} t_{\beta l} + \frac{\partial}{\partial x_K}\left(S_{Kl\beta\sigma}\frac{\partial w_\beta}{\partial x_\sigma}\right) = \rho \ddot{w}_l \qquad (29)$$

cancel

Equation (29), equations of motion, significantly reduce to

$$\frac{\partial}{\partial x_K}\left(S_{Kl\beta\sigma}\frac{\partial w_\beta}{\partial x_\sigma}\right) + \frac{\partial^2 w_l}{\partial x_K \partial x_\beta} t_{\beta K} = \rho \ddot{w}_l \qquad (30)$$

Recall $\frac{\partial t_{\beta K}}{\partial x_K} = 0$, hence

$$t_{\beta K}\frac{\partial^2 w_l}{\partial x_K \partial x_\beta} = \frac{\partial}{\partial x_K}\left(\frac{\partial w_l}{\partial x_\beta} t_{\beta K}\right) = \frac{\partial}{\partial x_K}\left(\frac{\partial w_\beta}{\partial x_\sigma} t_{\sigma K}\delta_{\beta l}\right) \qquad (31)$$

Substituting (31) into (30) and rearranging

$$\frac{\partial}{\partial x_K}\left\{\left(S_{Kl\beta\sigma} + t_{\sigma K}\delta_{l\beta}\right)\frac{\partial w_\beta}{\partial x_\sigma}\right\} = \rho \ddot{w}_l \qquad (32)$$

Equation (32) represents the dynamic equations of motion for a small deformation, w_i, superposed on a large deformation, u_i.

If the initial deformation is uniform (constant), then ρ_o, ρ, E_{mn}, t_{KL}, and $\partial x_K/\partial Y_m$ are all constants, hence $s_{K\ell\beta\sigma}$ is constant and (32) reduces to

$$\left(s_{K\ell\beta\sigma} + t_{\sigma K}\,\delta_{\ell\beta}\right) \frac{\partial^2 \omega_\beta}{\partial x_K \partial x_\sigma} = \rho \ddot{\omega}_\ell \qquad (33)$$

Assume displacements, ω_ℓ, are plane periodic waves

$$\omega_\ell = A_\ell e^{i(K_\ell x_\ell - \omega t)} = A_\ell e^{iK(\nu_\ell x_\ell - v t)} \qquad (34)$$

which are solutions to (33) and ν_ℓ is the propagation direction then substituting this solution into (33) yields

$$\left\{ \left(s_{K\ell\beta\sigma} + t_{\sigma K}\,\delta_{\ell\beta}\right) \nu_K \nu_\sigma - \rho v^2 \delta_{\ell\beta} \right\} A_\beta = 0 \qquad (35)$$

which is similar to Christoffel's equation but include an applied stress state, $t_{\sigma K}$, and $s_{K\ell\beta\sigma}$ that are functions of fourth order stiffness tensors, $C_{ijk\ell}$, and sixth order tensor terms, $C_{ijk\ell mn}$, that are related to the strain cubed terms of the strain energy density function, Σ, as follows using different indices. Starting with the strain energy density function, Σ, and keeping only strain squared and strain cubed terms

$$\Sigma(\underset{\sim}{E}) = \frac{1}{2} C_{rspq} E_{rs} E_{pq} + \frac{1}{6} C_{rspquv} E_{rs} E_{pq} E_{uv} + \text{H.O.} \qquad (36)$$

recall $$s_{n\ell ij} = \frac{\rho}{\rho_o} \frac{\partial^2 \Sigma}{\partial E_{rs} \partial E_{pq}} \frac{\partial x_i}{\partial Y_r} \frac{\partial x_j}{\partial Y_s} \frac{\partial x_k}{\partial Y_p} \frac{\partial x_m}{\partial Y_q} \qquad (37)$$

where $\dfrac{\partial^2 \Sigma}{\partial E_{rs} \partial E_{pq}} = C_{rspq} + C_{rspquv} E_{uv}$ and substituting into (37)

$$s_{n\ell ij} = \frac{\rho}{\rho_o} \left[C_{rspq} + C_{rspquv} E_{uv} \right] \frac{\partial x_n}{\partial Y_r} \frac{\partial x_\ell}{\partial Y_s} \frac{\partial x_i}{\partial Y_p} \frac{\partial x_j}{\partial Y_q} \qquad (38)$$

Recall $x_n = Y_n + u_n$, therefore $\partial x_n/\partial Y_r = \delta_{nr} + \dfrac{\partial u_n}{\partial Y_r} \cong \delta_{nr} + E_{nr}$ (39)
when deformations are homogeneous ("uniform") local rigid body rotations are not present and assume $\rho/\rho_o \approx 1$, then equation (38) reduces

$$S_{nlij} = \left[C_{rspq} + C_{rspquv} E_{uv} \right] (\delta_{nr} + E_{nr})(\delta_{ls} + E_{ls}) \cdot (\delta_{ip} + E_{ip})(\delta_{jq} + E_{jq}) \qquad (40)$$

Expand and keep only order E_{ij} terms

$$S_{nlij} = \left[C_{rspq} + C_{rspquv} E_{uv} \right] \left[\delta_{nr}\delta_{ls}\delta_{ip}\delta_{jq} + E_{nr}\delta_{ls}\delta_{ip}\delta_{jq} + E_{ls}\delta_{nr}\delta_{ip}\delta_{jq} + E_{ip}\delta_{nr}\delta_{ls}\delta_{jq} + E_{jq}\delta_{nr}\delta_{ls}\delta_{ip} \right] \qquad (41)$$

Equation (41) requires strains, E_{ij}, which can be obtained from

$$E_{ij} = C^{-1}_{ijkl} \, t_{kl} \qquad (42)$$

This is a reasonable approximation if the deformed state is small.

Procedure:

1) Determine: C_{ijkl}, C_{ijklmn}, t_{kl}, v_i
2) Invert stiffnesses: C^{-1}_{ijkl}
3) use (42) to calculate strains E_{ij}
4) use (41) to calculate S_{nlij}
5) substitute results into (35) and solve for eigenvalue wave velocities, v, and eigenvector particle displacement vibration orientations, A_β.
6) calculate effective modulus, $C^*_{nlij} = S_{nlij} + t_{nj}\delta_{li}$ and substitute into energy propagation direction

$$E_l = - C^*_{nlij} \, u_{i,j} \, \dot{u}_n \qquad (43)$$

Hence the energy propagation direction, E_l, can be steered using stress, t_{nj}, induced anisotropy.

Note: determining sixth order tensor components C_{rspquv} proves to be the most difficult.

Example: Unidirectional Graphite/Epoxy (transversely isotropic)

Material symmetry	Number of independent C_{rspquv}
triclinic	56
orthorhombic	20
hexgonal ("transversely isotropic")	9 see Ref. 6 below

Prosser/Kriz/Fitting Ref. 6: Experimental Stiffnesses & 3rd order constants

$C_{\alpha\beta}$ (GPa) 1 msi = 6.894 GPa

C_{11} (14.26), C_{12} (6.78), C_{13} (6.5)
C_{33} (108.4), C_{44} (5.27)

$C_{\alpha\beta\gamma}$ (GPa)

C_{111} (−196), C_{112} (−89), C_{113} (−4)
C_{123} (+65), C_{144} (−33.4), C_{155} (−49.1)
C_{344} (−47), C_{133} (−236), C_{333} (−829)

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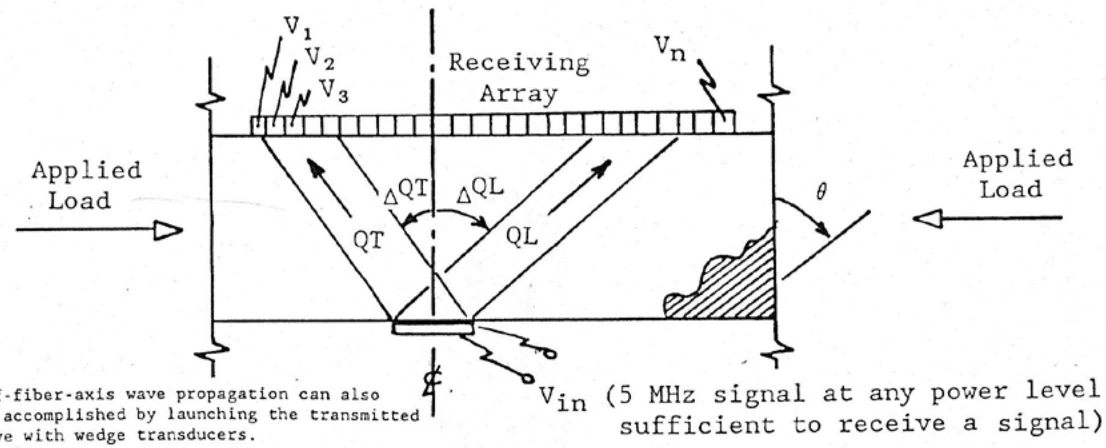
INVENTION DESCRIPTION

Inventors: Ronald D. Kriz, William H. Prosser,* and Dale W. Fitting

Device Objective: To monitor the amount of mechanical load applied to a graphite fiber-reinforced composite material.

Physics: Energy flux of stress waves propagating through anisotropic crystals has been shown to deviate from the direction of the normal to a plane wave. Graphite fiber-reinforced materials although not crystals are highly anisotropic and have been shown by the inventors to exhibit the same energy deviation phenomena in a predictable manner.

Device Operation: The device can be an integral part of the airframe where a unidirectional section of the structure (i.e. spar caps and longerons) is carrying a primary load.



*Off-fiber-axis wave propagation can also be accomplished by launching the transmitted wave with wedge transducers.

Figure 1. Schematic of load monitoring device*

United States Patent [19]

Prosser et al.

[11] **Patent Number: 5,209,123**

[45] **Date of Patent: May 11, 1993**

[54] METHODS OF DETERMINING LOADS AND FIBER ORIENTATIONS IN ANISOTROPIC NON-CRYSTALLINE MATERIALS USING ENERGY FLUX DEVIATION

[75] Inventors: **William H. Prosser**, Newport News; **Ronald D. Kriz**, Blacksburg, both of Va.; **Dale W. Fitting**, Golden, Colo.

[73] Assignee: **The United States of America as represented by the Administrator of the National Aeronautics and Space Administration, Washington, D.C.**

[21] Appl. No.: **797,507**

[22] Filed: **Nov. 22, 1991**

[51] **Int. Cl.**[5] **G01N 3/00**

[52] **U.S. Cl.** **73/788**; 73/641

[58] **Field of Search** 73/862.59, 778, 641, 73/599, 643, 633, 596, 788

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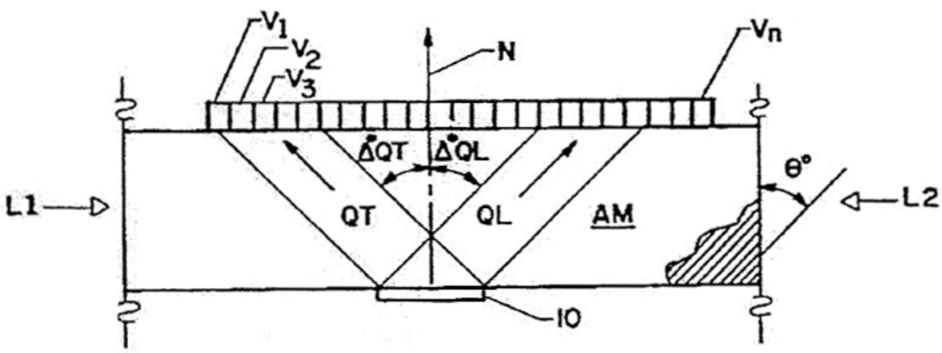
[57] **ABSTRACT**

An ultrasonic wave is applied to an anisotropic sample material in an initial direction and an angle of flux deviation of the ultrasonic wave front is measured from this initial direction. This flux deviation angle is induced by the unknown applied load. The flux shift is determined between this flux deviation angle and a previously determined angle of flux deviation of an ultrasonic wave applied to a similar anisotropic reference material under an initial known load condition. This determined flux shift is then compared to a plurality of flux shifts of a similarly tested, similar anisotropic reference material under a plurality of respective, known load conditions, whereby the load applied to the particular anisotropic sample material is determined. A related method is disclosed for determining the fiber orientation from known loads and a determined flux shift.

14 Claims, 3 Drawing Sheets



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Reply to Attn of 231 March 9, 1990

Dr. Ron Kriz
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Dear Ron,

I spoke with Dale Fitting yesterday and he said that he had mistakenly held onto the additional documents for the invention disclosure that I called you earlier this week about. He said that he would send those to me. When those arrive I will forward the invention disclosure on to the patent attorneys and inquire about when we might begin to present this work. I was thinking that, if they approve, we might consider submitting the work on the model calculations to this years QNDE conference. That is, unless you feel there is another conference or journal where this work should be presented. Since the deadline for abstract submission to QNDE is April 22, let me know soon what you think about this.

I also wanted to let you know that I am planning on visiting VPI for the annual fiber optics review meeting (Dr. Rick Clauss) with a group from Langley. I thought that this would be a good opportunity for us to get together and discuss our work. I believe the dates are April 10-12, 1990 although our travel plans are not finalized yet. Again, let me know of your schedule and your desire to get together at that time.

As for your comments the other day about doing this work with Mathematica, I am interested in using that program and have access to it here on our Mac II's. I made the following model calculations using Asyst because it was much easier to simply modify existing programs than write new ones. However, I can easily get the data into simple text files (as I did when I transferred the data to the Mac for plotting) and can thus send you disk copies of any of this data that you might desire. If you write any routines in Mathematica for this work, please send me a copy.

As for the model calculations on the effect of stress on energy flux deviation, I have done some additional work and plots of the results are enclosed. First, after some discussion with Dr. Bill Winfree, I was able to solve the problem of the sign of the energy flux deviation. It was so simple that I am almost embarrassed to tell you. I was computing the angle of the energy flux deviation by taking the arc cos of the dot product of the normalized energy flux vector and the wave normal vector which of course yields the correct angle between the two vectors. However, this does not tell whether one vector is at a positive or negative angle with respect to the other. Since we are working on wave propagation within a plane, the problem is solved by using the arc tan to determine the angle of each vector with respect to the coordinate axes and then subtracting to find the angle between the vectors including the proper sign. This was implemented in the model program and the revised chart of flux deviations with the proper sign is given on the first enclosure.

Next, I modified the program to make it sweep through the calculations of the flux deviation as a function of the angle of the fiber orientation. The actual points for these plots are spaced at two degree intervals. This yields plots similar to those in your paper on the effects of moisture on flux deviation. The first plot (fig. 2) shows the pure mode transverse wave for zero stress, 1 GPa uniaxial stress along the fibers and 0.1 GPa uniaxial stress perpendicular to the fibers. The next plot (3) shows the change in flux deviation for the 2 stress cases for the PT wave as a function of fiber orientation. It shows a peak in "sensitivity" of about 60 deg. fiber orientation for this wave mode.

The next plots are the same type except for showing the effect of stress on the quasi-longitudinal and quasi-transverse waves. Fig. 4 is very similar to your plot and shows the cross over of the QL and QT waves at around 74 degrees. The next plot (5) shows the flux shift for the QL and QT waves for the two stress states. The large peaks that appear at the cross over angle are an artifact of the cross over as is documented in several of the later plots. However, there are larger shifts before and after the cross over. Plot 6 shows the flux shift over the 10 to 60 degree range which shows the maximum sensitivity to be in the QT wave at around 22 degrees.

The next few plots show the transition region for the QL and QT waves. The program was again modified to increase the resolution so that 45 points were calculated over the 70 to 80 degree range. Plot 7 shows the behavior of these waves with the common plot symbols following the flux deviation for a particular e-value. Whether it was actually the QL or the QT wave was not actually calculated. However, from your earlier work, I believe the solid points are the QT wave up until 4 points before the cross over where the open points become QT. The reverse would then be true for the QL wave. If this is the case, then the actual QL and QT wave curves would be plotted as in plot 8. It is interesting that this discontinuity occurs and this was the cause of the early anomalous behavior that was pointed out in the flux shift curves. They were calculated following a particular e-value, not for the proper QL or QT wave. This behavior was even more interesting after the application of load in the model. Plot 9 shows the influence of 1 GPa stress along the fiber direction on this transition region. Note that the actual crossing point of the two curves shifts somewhat. In addition, if we make the same assumption that I did previously that the

QL and QT surfaces make the transition before the curve, the QL and QT curves are no longer continuous as plotted in Fig. 10. This probably does not make any difference in the basic scheme of life as we know it, but I found it to be interesting behavior and I thought you might be interested in it as well.

The final question that I investigated was whether the effect of stress would produce a linear change in flux shift. It seemed natural that it would and plots 11 and 12 show that the model confirms this. Fig. 11 shows the flux shift for the three modes as uniaxial stress is applied along the fibers while 12 shows the same as the stress is applied perpendicular to the fibers. In both cases, the wave was considered to be propagating at 45 degrees with respect to the fibers.

In closing, I've probably given you more information than you cared to, or had time to examine, but I hope you find some of it interesting. Let me know any comments you have on this work, or on the presentation of it, and my proposed visit.

Sincerely,

Bill Prosser

William H. Prosser
Aerospace Technologist, Nondestructive Measurement Sciences Branch, IRD,
NASA Langley Research Center

Table 1. ①

Energy flux deviation angles (delta in degrees)

	Wave Mode	Stress State: 0 Stress	0.5 GPa along x3	1.0 GPa along x3	0.1 GPa along x1
30 deg wrt x3	QL	- 27.96	- 28.01	- 28.05	- 28.02
	QT	24.55	25.98	27.32	23.29
	PT	- 7.72	- 7.97	- 8.22	- 7.80
45 deg wrt x3	QL	- 41.39	- 41.48	- 41.55	- 41.51
	QT	22.03	23.02	23.96	21.15
	PT	- 9.64	- 9.97	- 10.31	- 9.75
60 deg wrt x3	QL	- 53.17	- 53.36	- 53.53	- 53.47
	QT	12.64	13.25	13.91	12.49
	PT	- 9.13	- 9.48	- 9.83	- 9.25

Pure transverse wave

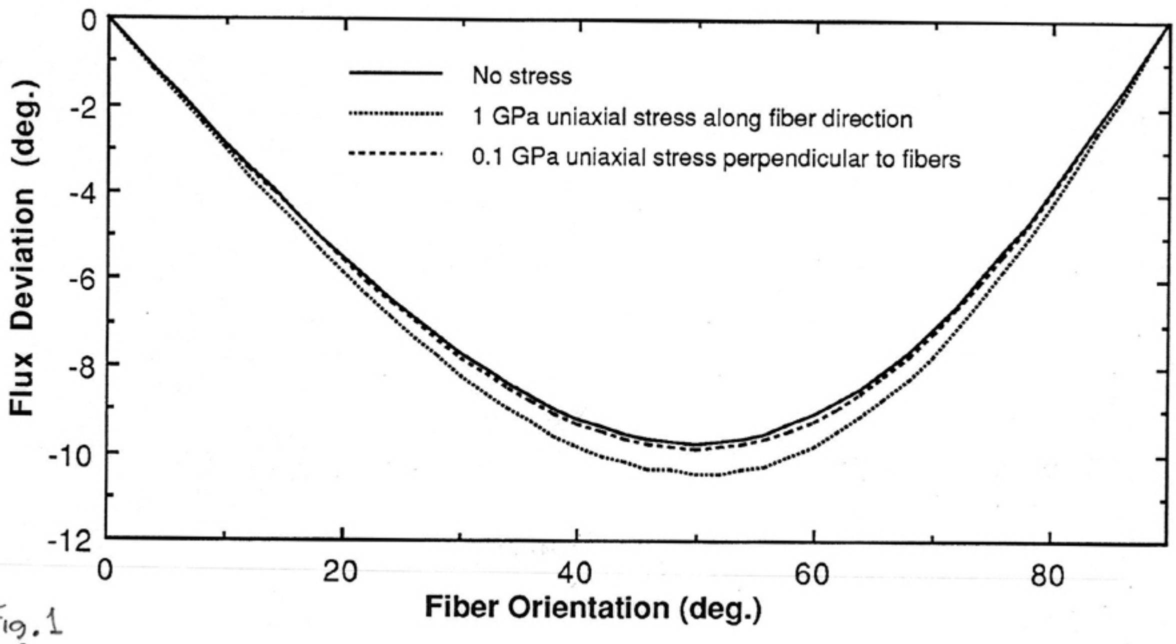

Fig. 1 ②

Pure transverse wave

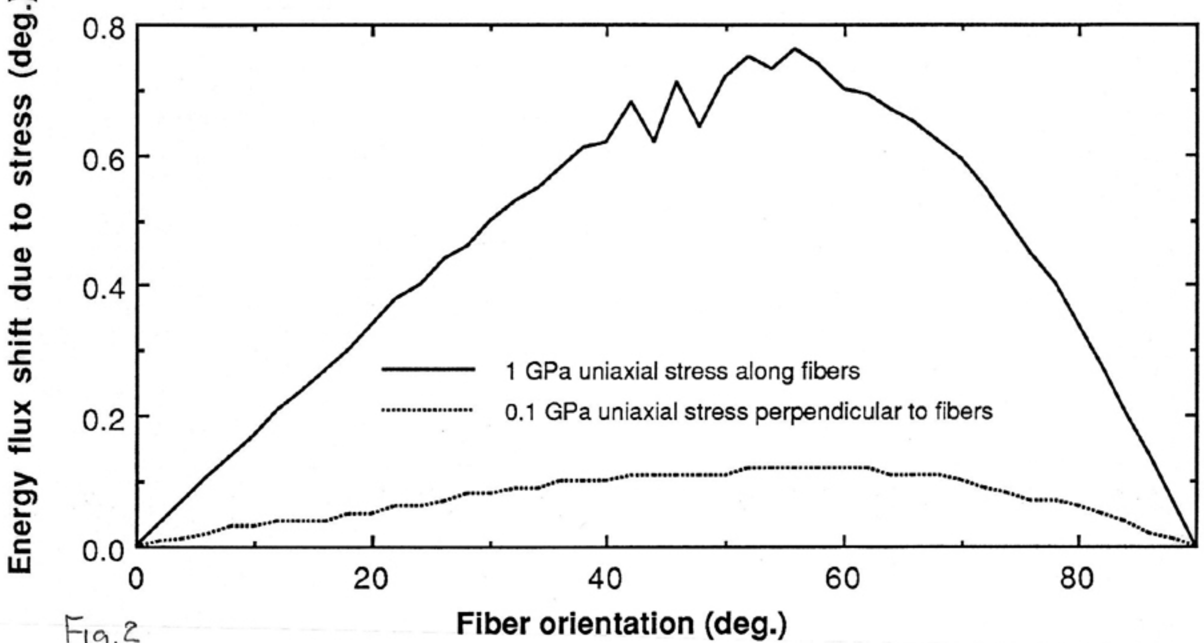

Fig. 2
③

Quasilongitudinal and Quasitransverse waves

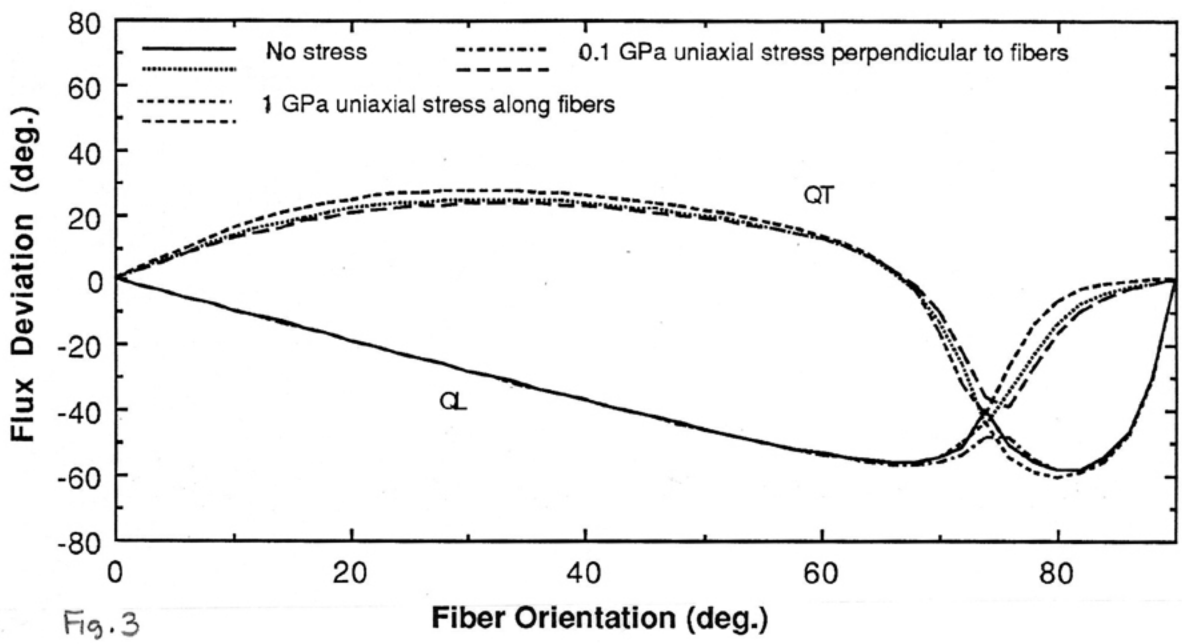

Fig. 3
④

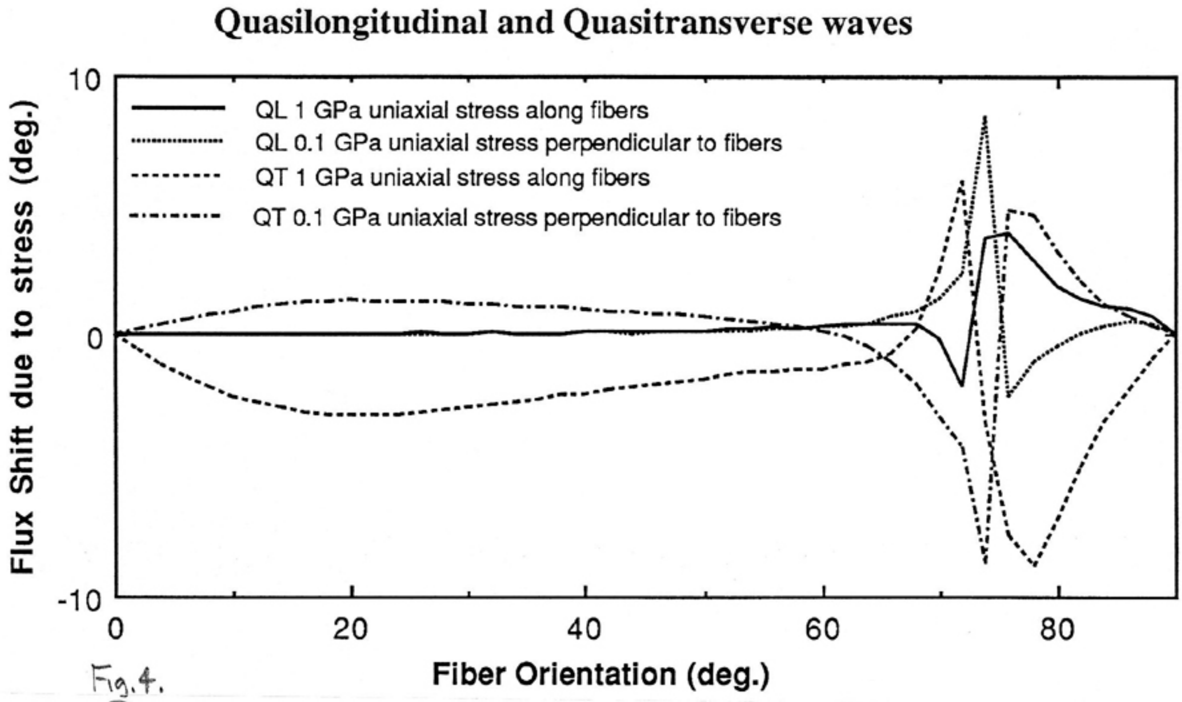
Quasilongitudinal and Quasitransverse waves
QL 1 GPa uniaxial stress along fibers
QL 0.1 GPa uniaxial stress perpendicular to fibers
QT 1 GPa uniaxial stress along fibers
QT 0.1 GPa uniaxial stress perpendicular to fibers
Flux Shift due to stress (deg.)
Fiber Orientation (deg.)
10
0
-10
0
20
40
60
80

Fig. 4.
(5)

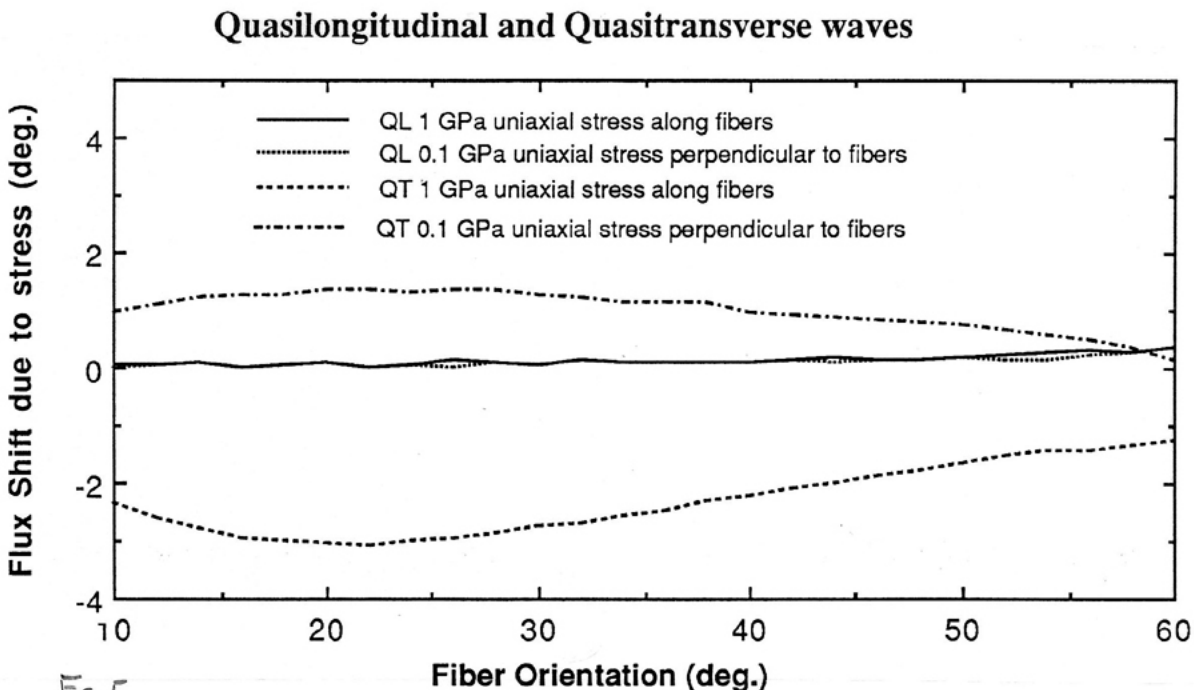
Quasilongitudinal and Quasitransverse waves
QL 1 GPa uniaxial stress along fibers
QL 0.1 GPa uniaxial stress perpendicular to fibers
QT 1 GPa uniaxial stress along fibers
QT 0.1 GPa uniaxial stress perpendicular to fibers
Flux Shift due to stress (deg.)
Fiber Orientation (deg.)
4
2
0
-2
-4
10
20
30
40
50
60

Fig. 5
(6)

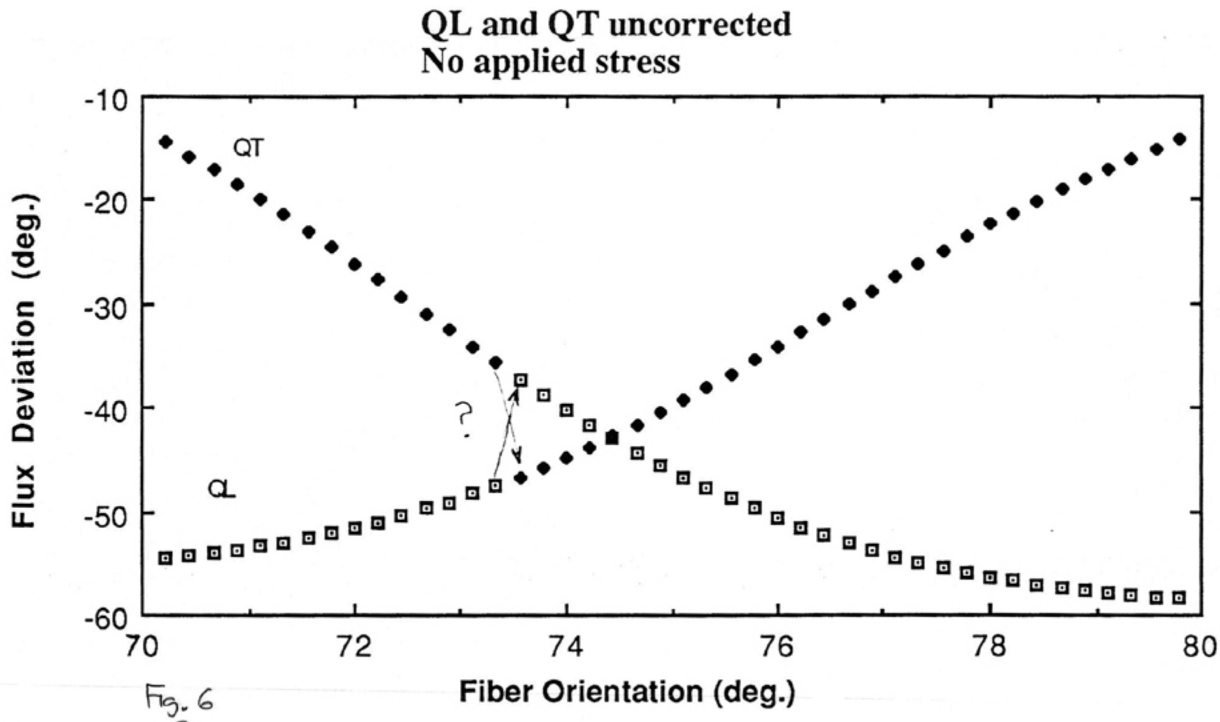
QL and QT uncorrected
No applied stress
Flux Deviation (deg.)
Fiber Orientation (deg.)
QT
QL
?
-10
-20
-30
-40
-50
-60
70
72
74
76
78
80

Fig. 6
(7)

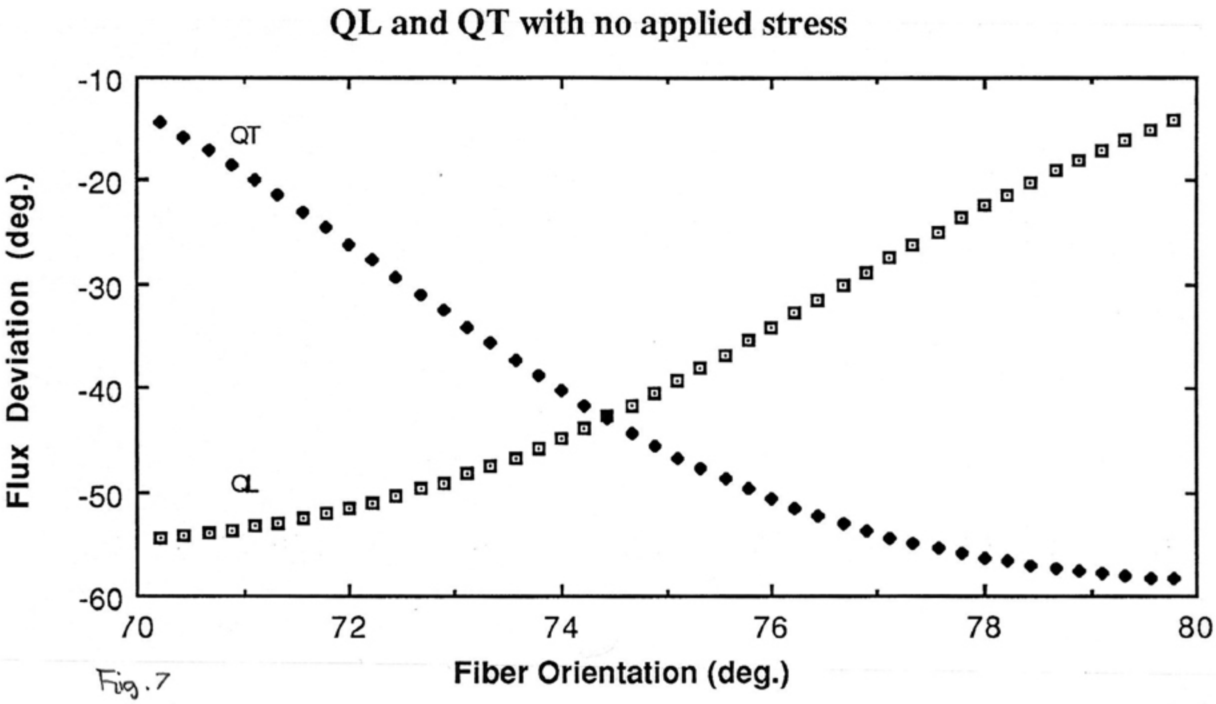
QL and QT with no applied stress
Flux Deviation (deg.)
Fiber Orientation (deg.)
QT
QL
-10
-20
-30
-40
-50
-60
70
72
74
76
78
80

Fig. 7
(8)

QL and QT uncorrected
1 GPa uniaxial stress along fibers

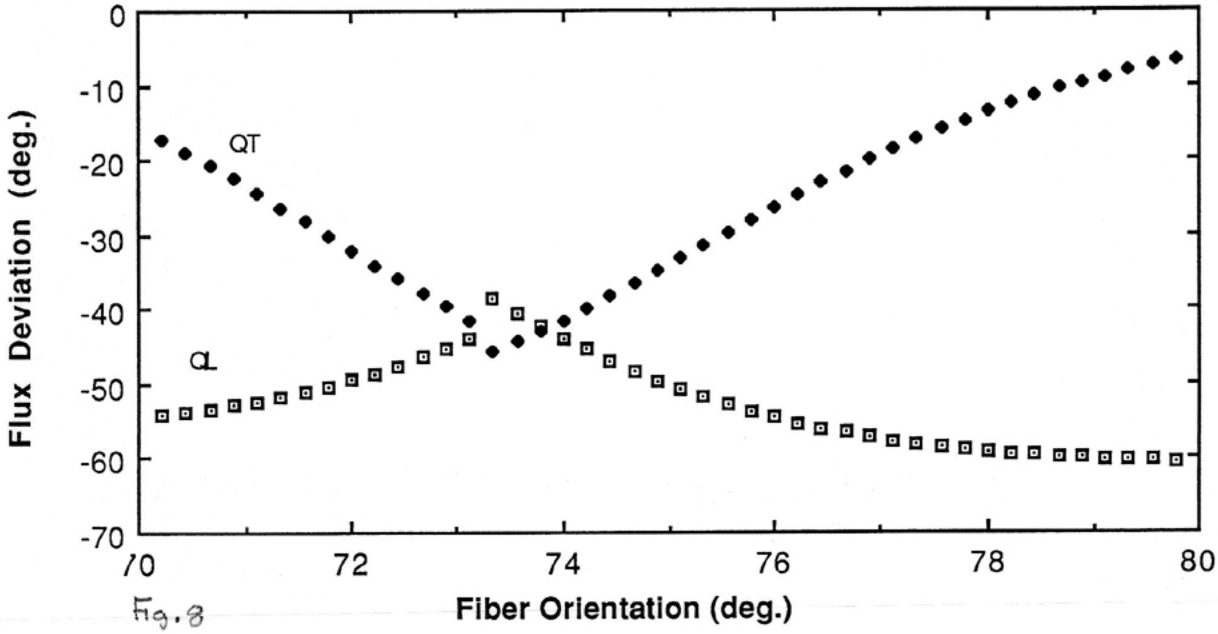

Fig. 8
(9)

QL and QT
1 GPa uniaxial stress along fibers

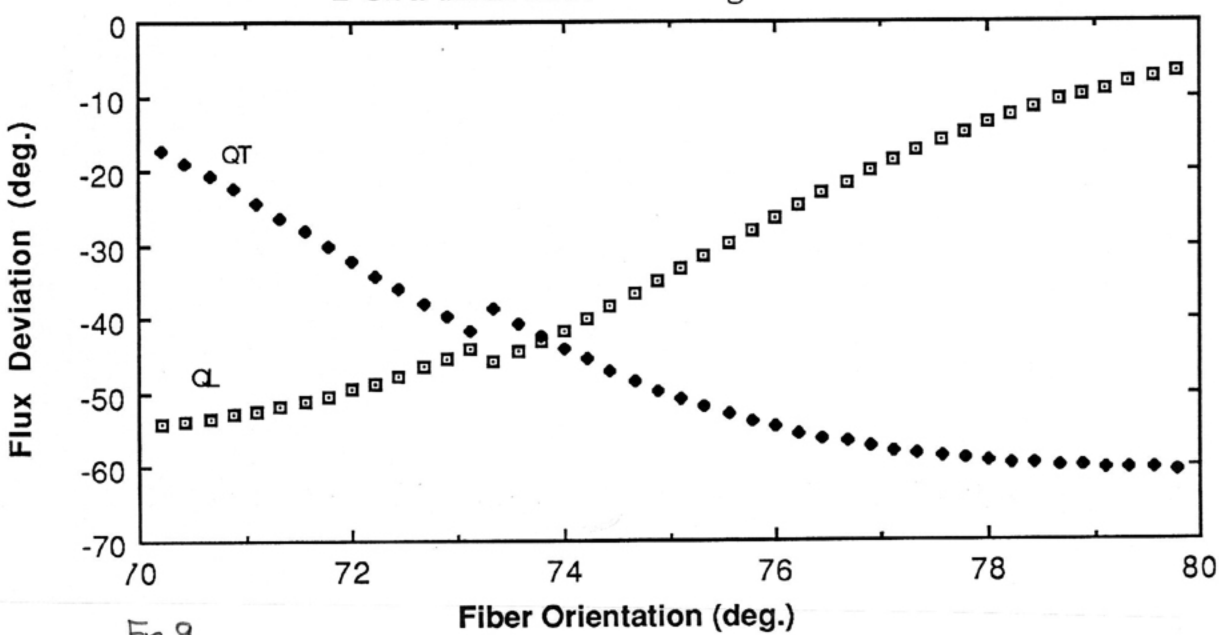

Fig. 9
(10)

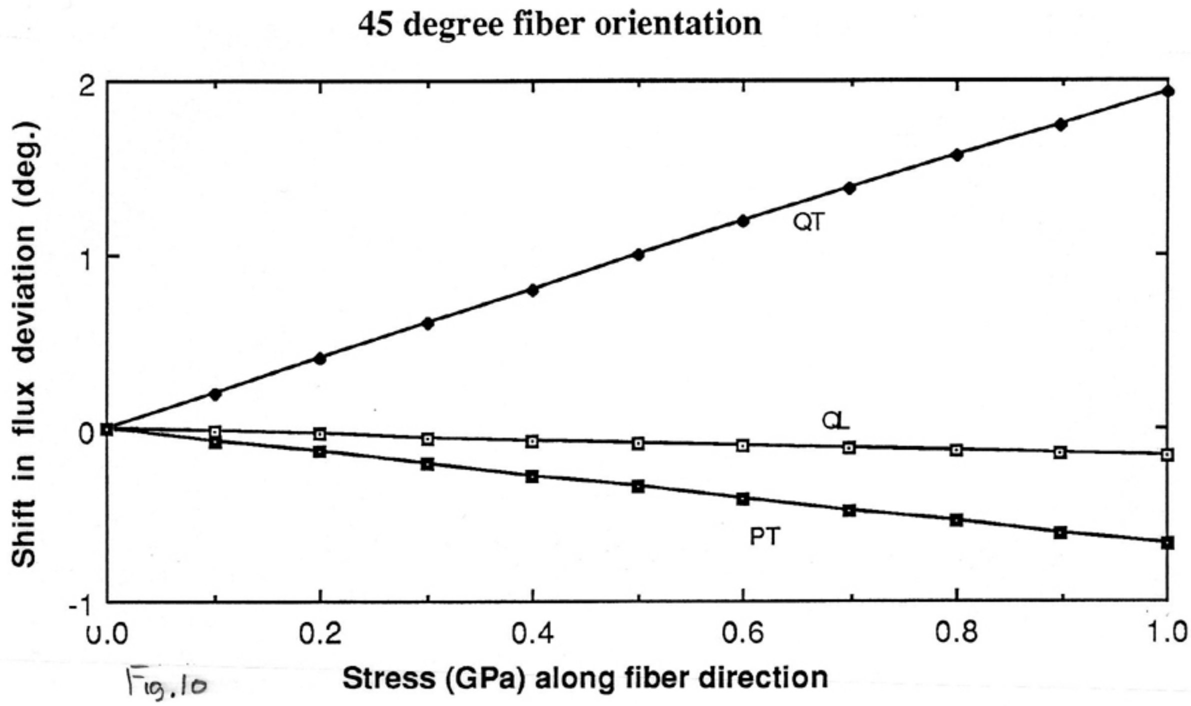

Fig. 10

(11)

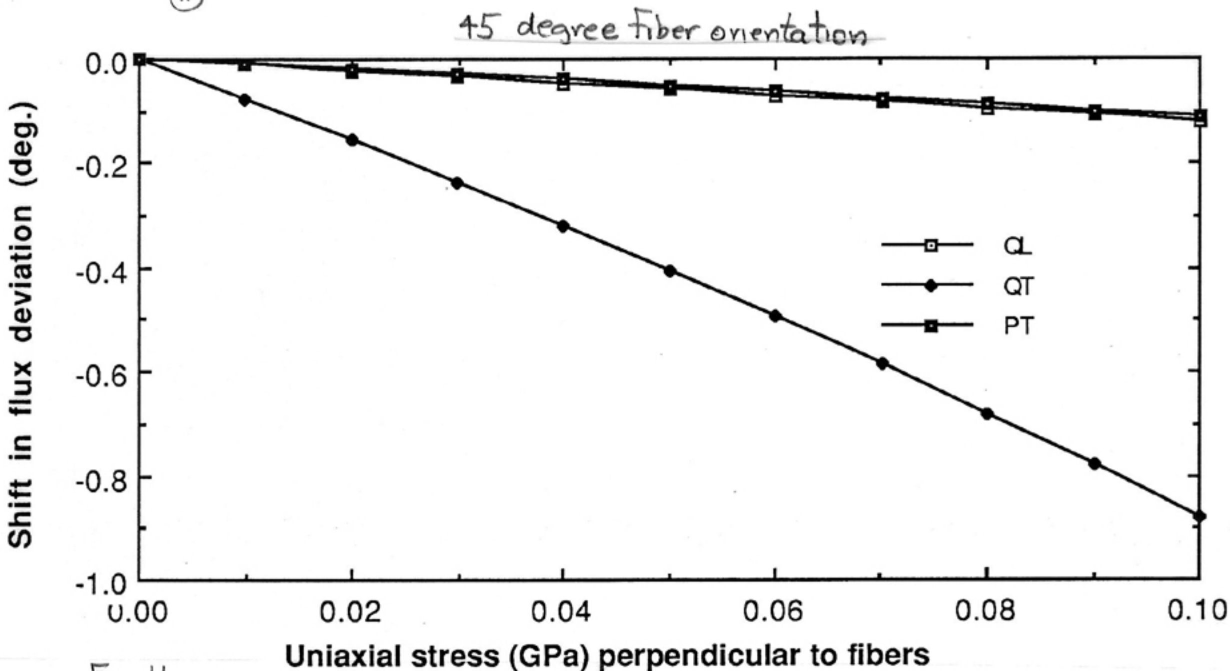

Fig. 11

(12)