$\delta = \frac{PL}{AE}$, $\theta = \frac{TL}{JG}$, $J = \frac{\pi r^4}{2}$, $\tau^{max} = \frac{Tc}{J}$

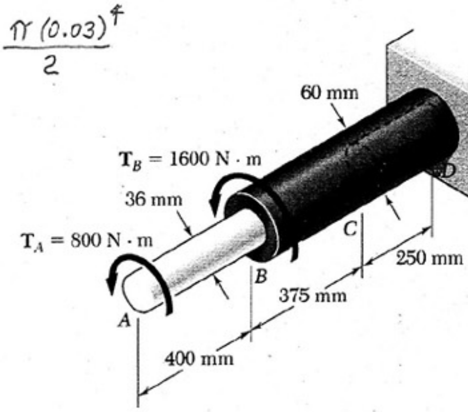
MULTIPLE CHOICE PROBLEMS (10 Points each)

1. The aluminum rod *AB* (*G* =27 GPa) is bonded to the brass rod *BD* (*G* = 39 GPa). Knowing that portion *CD* of the brass rod is hollow and has an inner diameter of 40 mm, the angle of twist at *A* is most nearly:

$J_{AB} = \frac{\pi (0.018)^4}{2}$, $J_{BC} = \frac{\pi (0.03)^4}{2}$

$J_{CD} = \frac{\pi}{2}\left[(0.03)^4 - (0.02)^4\right]$

(a) 45.3×10^{-3} rad
(b) 105×10^{-3} rad
(c) 71.9×10^{-3} rad
(d) 15.07×10^{-3} rad
(e) 126.4×10^{-3} rad



$$\theta_A = \frac{T_{AB} L_{AB}}{J_{AB} G_{AB}} + \frac{T_{BC} L_{BC}}{J_{BC} G_{BC}} + \frac{T_{CD} L_{CD}}{J_{CD} G_{CD}}$$

$$\frac{2\cdot 800\,(0.4)}{(0.018)^4 \pi (27\times10^9)} + \frac{2\cdot 2400(0.375)}{(0.03)^4 \pi (39\times10^9)} + \frac{2\cdot 2400\,(0.25)}{\left[(0.03)^4-(0.02)^4\right]\pi(39\times10^9)}$$

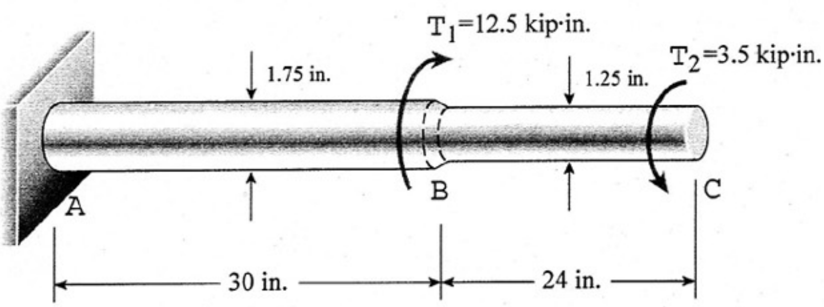
10.5×10^{-8} 81×10^{-8} $[81\times10^{-8} - 16\times10^{-8}]$ 65×10^{-8}

$$\frac{1}{\pi}\left\{0.2258 + 0.05698 + 0.04734\right\}$$

$$= 0.33/\pi = \boxed{0.105 \text{ rads}} = \theta_A$$

2. For the solid brass shaft shown, G = 17×10^3 ksi. The maximum shear stress in the shaft is most nearly

(a) 8.55 ksi
(b) 9.13 ksi
(c) 11.88 ksi
(d) 32.6 ksi
(e) 4.33 ksi



$J = \frac{\pi r^4}{2}$

$\tau^{max} = \frac{Tc}{J}$

$T_{AB} = 9$ Kip in , $T_{BC} = 3.5$ Kip-in

$J_{AB} = \frac{\pi (0.875)^4}{2} = 0.9208$, $J_{BC} = \frac{\pi (0.625)^4}{2} = 0.2397$

$C_{AB} = 0.875$ $C_{BC} = 0.625$

$\tau^{max}_{AB} = \frac{9\,(0.875)}{0.9208}$ $\tau^{max}_{BC} = \frac{3.5\,(0.625)}{0.2397}$

$= 8.55$ Ksi

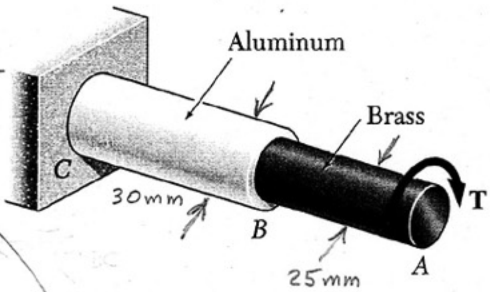
$\tau^{max}_{BC} = \boxed{9.13 \text{ Ksi}}$ ✓

Largest

$$\tau^{max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi c^4}{2}} = \frac{2T}{\pi c^3}$$

3. The *solid* rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made of brass for which the allowable shearing stress is 50 MPa. The largest inner diameter of rod *AB* for which the factor of safety is the same for each rod is most nearly

(a) 16.13 mm
(b) 12.50 mm
(c) 9.61 mm
(d) 11.22 mm
(e) 15.18 mm ✓



$\tau^{all}_{AB} = 50\,MPa$; $\tau^{all}_{BC} = 25\,MPa$

$F.S.^{AB} = F.S.^{BC}$

$$\tau^{max}_{AB}/\tau^{all}_{AB} = \tau^{max}_{BC}/\tau^{all}_{BC}$$

$$\tau^{max}_{AB} = \frac{T\,c^{o}_{AB}}{\frac{\pi}{2}\left[c^{o\,4}_{AB} - c^{i\,4}_{AB}\right]} \qquad \tau^{max}_{BC} = \frac{T\,c_{BC}}{\frac{\pi}{2}\,c^4_{BC}}$$

$$\frac{\cancel{T}\,(0.0125)}{(50)\cancel{\frac{\pi}{2}}\left[(0.0125)^4 - c^{i\,4}_{AB}\right]} = \frac{\cancel{T}\,\cancel{(0.015)}}{\cancel{\frac{\pi}{2}}\,(0.015)^{\cancel{4}\,3}\,(25)}$$

$$(25)(0.015)^3(0.0125) = \left[(0.0125)^4 - c^{i\,4}_{AB}\right](50)$$

$$c^{i\,4}_{AB} = 0.0125^4 - (0.015)^3(0.0125)/2 = 24.41\times10^{-9} - 21.09\times10^{-9} = 3.32\times10^{-9}\,m^4 = c^{i\,4}_{AB}$$

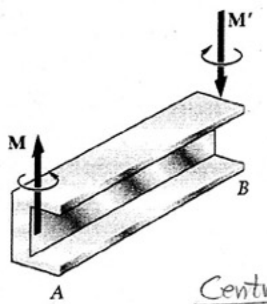
$c^{i}_{AB} = 0.00759\,m$

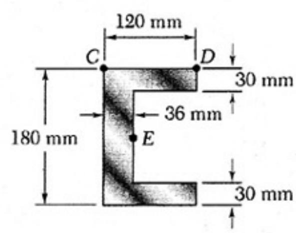
$c^{i}_{AB} = 7.59\,mm$

$$\boxed{d^{i}_{AB} = 15.18\,mm}$$

4. Two equal and opposite couples of magnitude $M = 25$ kN·m are applied to the channel-shaped beam *AB*. Observing that the couples cause the beam to bend in the horizontal plane, the normal stress at point *C* is most nearly:

(a) 79.8 MPa tension ✓
(b) 79.8 MPa compression
(c) 136.5 MPa tension
(d) 136.5 MPa compression
(e) 100.7 MPa tension





$$\sigma = +\frac{My}{I} = +\frac{25\bar{Y}}{I}$$

$$\sigma = \frac{25\times10^3\,(0.04425)}{13.869\times10^{-6}}$$

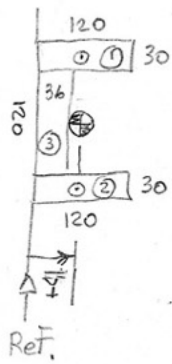
$$\boxed{\sigma = 79.76\times10^6\,Pa}$$

Centroid

	y	A	yA
1	60	3600	216000
2	60	3600	216000
3	18	4320	77760
		11520	509760

$= \bar{Y}$

$\bar{Y} = 44.25$



Moment Inertia

$$I = 2I_{①} + I_3 = 13.869\times10^6\,mm^4 = 13.869\times10^{-6}$$

$$1\,m = 1000\,mm \qquad \frac{1\,m^4}{1.0\times10^{12}\,mm^4}$$

$$I_{②} = \bar{I}_{①} = I_{①} + Ad^2_{①} = \frac{30\,(120)^3}{12} + (120)(30)\left[60 - 44.25\right]^2$$

$$= 4.32\times10^6 + 0.893\times10^6 = 5.213\times10^6 = I_{①} = I_{②}$$

$\bar{I}_{③} = 3.443\times10^6$

$$\bar{I}_{③} = I_{③} + Ad^2_{③} = \frac{120\,(36)^3}{12} + (120)(36)\left[44.25 - 18\right]^2 = 0.4666 + 2.9768$$

5. A vertical force **P** of magnitude 20 kips is applied at a point C located on the axis of symmetry of the cross section of a short column. This point is located at $y = 5$ in. from the bottom of the column, as shown. The centroid of the section is located $y = 3.8$ in. from the bottom of the column. The neutral axis location is most nearly at:

$A = A_{①} + A_{②} = 20\,in^2$
$12 + 8$

✓ (a) $y = 1.389$ in
~~(b) $y = 3.80$ in~~
(c) $y = 1.200$ in
(d) $y = 4.15$ in
(e) $y = 3.65$ in

$\bar{I} = I + A d^2$

$\bar{I}_{①} = I_{①} + A_{①} d_{①}^2$

$= \frac{bh^3}{12} + (bh)(5-3.8)^2$

$= \frac{6(2)^3}{12} + (2)(6)(1.2)^2$

$\bar{I}_{①} = 4 + 17.28 = 21.28\,in^4$

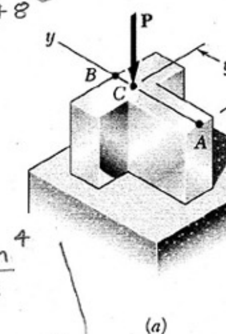
$\bar{I}_{②} = \frac{2(4)^3}{12} + (2)(4)[3.8-2]^2$

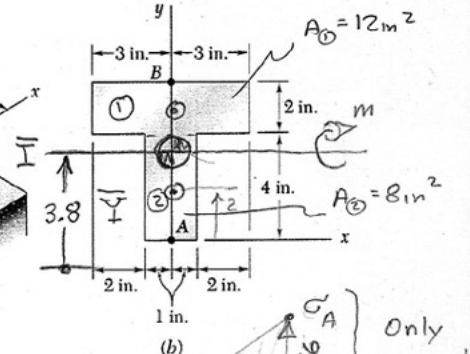
$\bar{I}_{②} = 10.667 + 25.92 = 36.59\,in^4$

$\bar{I} = \bar{I}_{①} + \bar{I}_{②}$

$\bar{I} = 57.87\,in^4$

$M = P(5-3.8)$
$= 20(1.2)$
$M = 24$ Kip-in





$$\sigma_A = -\frac{P}{A} + \frac{My}{I} = -\frac{20}{20} + \frac{24(3.8)}{57.87} = 0.576$$
$-1 + 1.576$

$$\sigma_B = -\frac{P}{A} - \frac{My}{I} = -\frac{20}{20} - \frac{24(2.2)}{57.87} = -1.912$$
$-1 - 0.912$

$\boxed{2.488}$

Similar triangles

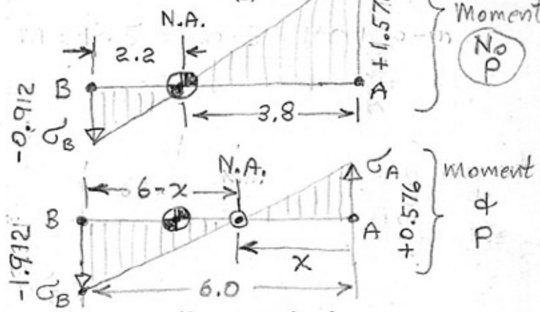
$$\frac{6-x}{|\sigma_B|} = \frac{x}{|\sigma_A|}$$

$6\sigma_A - x\sigma_A = x\sigma_B$

$6\sigma_A = x(\sigma_A + \sigma_B)$

$x = 6\sigma_A / (\sigma_A + \sigma_B)$

$x = \boxed{1.389\,in}$



6. A horizontal force **P** = 20 kips is applied to the beam shown. The maximum tensile stress in the beam is most nearly

✓ (?) 1.266 Ksi
(a) 0.933 ksi
(b) 1.634 ksi
(c) 2.16 ksi
(d) 0.417 ksi
✓ (e) 1.113 ksi
Closest

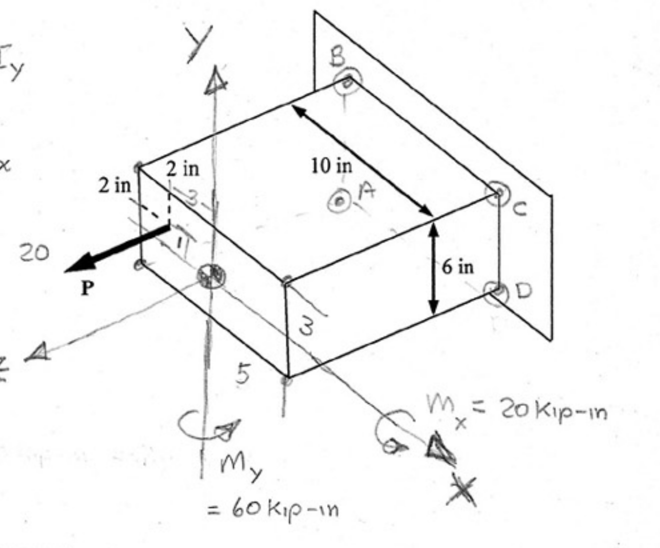
1.634
1.266
.368

1.266
1.113
.153

$I_y = \frac{bh^3}{12} = \frac{6(10)^3}{12} = 500 = I_y$

$I_x = \frac{bh^3}{12} = \frac{10(6)^3}{12} = 180 = I_x$

$A = 60\,in^2$



$$\sigma_A = +\frac{M_y\,5}{I_y} - \frac{M_x\,3}{I_x} + \frac{P}{A}$$

$$= +\frac{60(5)}{500} - \frac{20(3)}{180} + \frac{20}{60}$$
$+(0.333)$

$\sigma_A = +0.6 - 0.333 =$ small

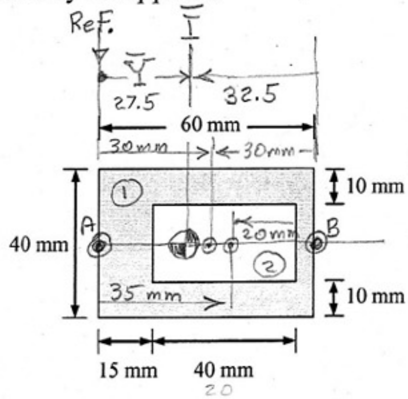
$\sigma_B = +0.6 + 0.333 = +0.933 + 0.333 = \boxed{1.266}$ ✓

$\sigma_C = -0.6 + 0.333 =$ small

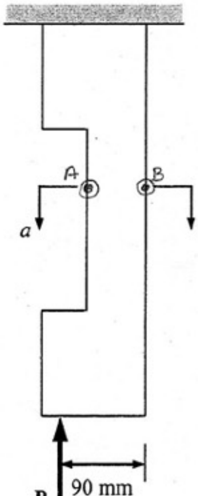
$\sigma_D = -0.6 - 0.333 = -0.933 + 0.333 = -0.633$

7. WORK OUT PROBLEM (40 Points) **Name:** ____________________

A load **P** is applied upward on the wooden which has a hollow cross-section in the middle portion as shown. Knowing that in Section *a-a* the allowable stresses are 35 MPa in compression and 25 MPa in tension, determine the largest allowable load **P** that may be applied.



section *a-a*



Locate Centroid: $\boxed{20/40}$ $A = 0.16 \times 10$

	x	A	xA
①	30	2400	72 000
②	35	-800	-28000
		1600	44000

$\frac{44000}{1600} = \boxed{\bar{Y} = 27.5}$

Bending Moment of Inertia: $\boxed{10/40}$

$\bar{I} = \bar{I}_{①} - \bar{I}_{②}$; $\bar{I}_{①} = I_{①} + A_{①} d_{①}^2$

$= 0.735 \times 10^{-6} m^4$ $\quad = \frac{bh^3}{12} + bh(30 - 27.5)^2$

$- 0.152 \times 10^{-6} m^4$ $\quad = \frac{40(60)^3}{12} + 2400(2.5)^2$

$\boxed{\bar{I} = 0.583 \times 10^{-6} m^4}$ $\quad = 720{,}000 + 15\,000 = 0.735 \times 10^{+6}\ mm^4$

$\left(\frac{1 m^4}{1 \times 10^{12} mm^4}\right)$ $\quad \bar{I}_{①} = 0.735 \times 10^{-6} m^4$

$A = 0.16 \times 10^4 mm^2$

$A = 0.16 \times 10^{-2} m^2$

$\bar{I}_{②} = I_{②} + A_{②} d_{②}^2 = \frac{bh^3}{12} + bh(35 - 27.5)^2$

$= \frac{20(40)^3}{12} + 800(7.5)^2 = 106\,667 + 45000 = 151\,667 mm^4 = 0.1517 \times 10^{-6} m^4 = \bar{I}_{②}$

Calculate P: $\boxed{10/40}$

Moment: $P(0.090 - 0.0325) = M = 0.0575\, P$ N-m

Allowables: $\sigma_{Ten}^{all} = 25 MPa$, $\sigma_{comp}^{all} = 35 MPa$

$\sigma_A = -\frac{P}{A} - \frac{M(0.0275)}{\bar{I}} = -35 \times 10^{+6}$, $\sigma_B = -\frac{P}{A} + \frac{M(0.0325)}{\bar{I}} = 25 \times 10^6$

$-\frac{P}{0.16 \times 10^{-2}} - \frac{0.158 \times 10^{-2} P}{0.583 \times 10^{-6}} = -35 \times 10^{+6}$, $-\frac{P}{0.16 \times 10^{-2}} + \frac{0.1869 \times 10^{-2} P}{0.583 \times 10^{-6}} = +25 \times 10^6$

$-3.644 \times 10^{-4} P - 15.8 \times 10^{-4} P = 20.41$ $\quad -3.644 \times 10^{-4} P + 18.69 \times 10^{-4} P = 14.575$

$19.44 \times 10^{-4} P = 20.41$ $\quad 15.05 \times 10^{-4} P = 14.575$

$P = 10{,}496 N$ $\quad \boxed{P = 9{,}684 N}$ ✓

Largest Allowed

Grade:

① Centroid: 20/40

② $\bar{I}$: 10/40

③ P : 10/40