**MULTIPLE CHOICE PROBLEMS (10 Points each)**
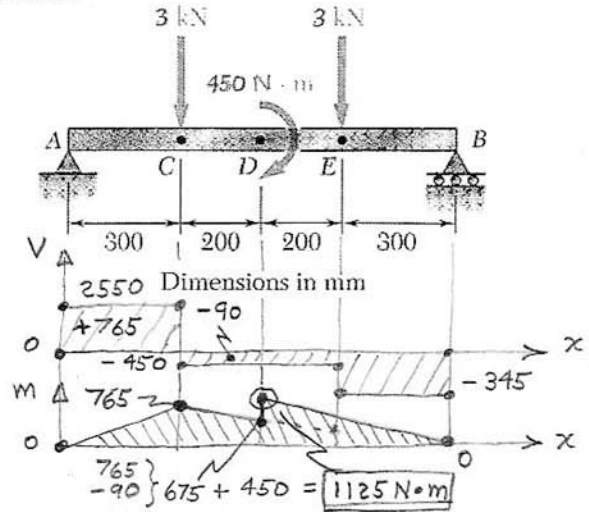
$\circlearrowleft \sum M@_B = 0 = -R_A \cdot 1 + 3000(0.7)^m$

$R_A = 2550\,N \uparrow \quad +3000(0.3)^m - 450\,N \cdot m$

$\underline{R_B = 3450\,N}$

1. Determine the maximum absolute value of the bending moment.

   a. 675 N•m
   b. 765 N•m
   c. 900 N•m
   d. 1125 N•m
   e. 3450 N•m



Dimensions in mm

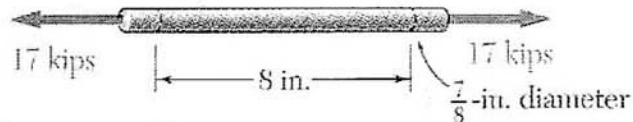$765 \atop -90 \} 675 + 450 = \boxed{1125\,N \cdot m}$

2. Determine the maximum shear stress in the steel rod.

   a. 32.1 ksi
   b. 8.50 ksi
   c. 14.14 ksi
   d. 0 ksi
   e. 28.3 ksi

$\sigma_x = P/A$

$= \dfrac{17\,kips}{\pi/4\,(7/8)^2}$

17 kips $\longleftarrow$ 8 in. $\longrightarrow$ 17 kips, $\frac{7}{8}$-in. diameter

$\sigma_x = 28.3\,ksi = \sigma_{max} \qquad \sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}|\sigma_{max} - \sigma_{min}| = \frac{1}{2}|28.3 - 0| \qquad \boxed{\tau_{max} = 14.14\,ksi}$

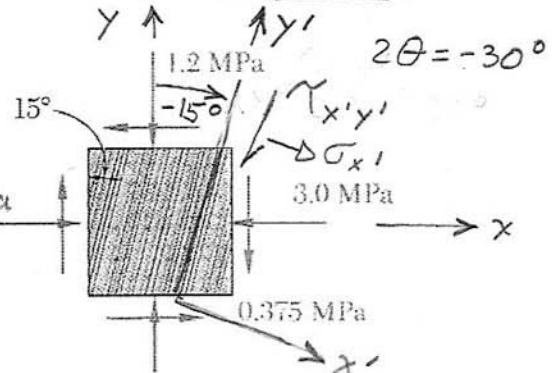3. Determine the magnitude of the in-plane shearing stress parallel to the grain (magnitude only), and the normal stress perpendicular to the grain.

   a. 0.775 MPa, -1.508 MPa
   b. 0.975 MPa, -2.692 MPa
   c. 0.775 MPa, -2.692 MPa
   d. 0.595 MPa, -0.772 MPa
   e. 0.595 MPa, -2.872 MPa

$\sigma_x = -3\,MPa$

$\sigma_y = -1.2\,MPa$

$\tau_{xy} = -0.375\,MPa$

$2\theta = -30°$



$\sigma_\perp = \sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$\text{grain} \qquad = \dfrac{-3-1.2}{2} + \dfrac{-3+1.2}{2}\cos(-30°) - 0.375\sin(-30°) = \underline{-2.692\,MPa}$

$\sigma_\parallel = \tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$\text{grain}$

$-\dfrac{-3+1.2}{2}\sin(-30°) - 0.375\cos(-30°) = \boxed{|0.775\,MPa|} \quad |\text{magnitude}|$

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4. A spherical pressure vessel is 3 m in diameter and has a wall thickness of 12 mm. Knowing that $\sigma_{all} = 80$ MPa, determine the maximum allowable gage pressure.

   a. 0.645 MPa
   b. 1.290 MPa
   c. 2.560 MPa
   d. 0.640 MPa
   e. 0.385 MPa

$r = d/2 - t = 1.5 - 0.012$

$r = 1.498$

$\sigma_2 = \dfrac{pr}{2t} = \sigma_{allow} = 80 \text{ MPa}$

$p = \dfrac{2(0.012)\,80}{1.498} = \boxed{1.282 \text{ MPa} = P}$

5. Three boards, each 50 mm thick, are nailed together to form a beam that is subject to a 1200 N vertical shear. Knowing that the nail spacing is 81.5 mm, determine the shear stress in each of the nails. The diameter of each nail is 5 mm. The value of I for the section is $91.67 \times 10^6$ mm$^4$.

   (a) 71.2 MPa
   (b) 30.6 MPa
   (c) 0.1473 MPa
   (d) 27.4 MPa
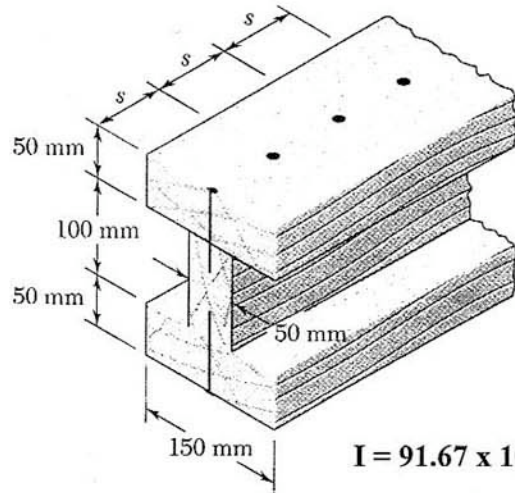   (e) 147.3 MPa

$q = \dfrac{VQ}{I}$

$F = q \cdot S = \dfrac{VQ}{I} \cdot S$

$F = \dfrac{1200\,(5.625 \times 10^5)\,81.5}{91.67 \times 10^6}$

$F = 600 \text{ N}$

$\tau = \dfrac{F}{A} = \dfrac{600 \text{ N}}{\pi/4 \,(0.005)^2}$

**I = 91.67 x 10⁶ mm⁴**

$Q = \bar{y}A \qquad A = 50\,(150)$

$\bar{y} = 75$

$Q = 75\,(50)150 = 5.625 \times 10^5 \text{ mm}^3$

$\boxed{\tau = 30.6 \text{ MPa}}$
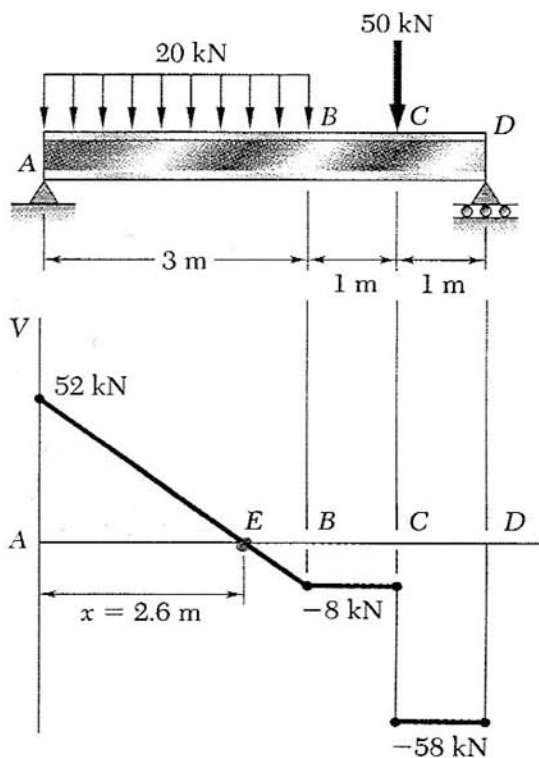
50 mm

100 mm

50 mm

50 mm

150 mm

6. A simply supported steel beam AD is to carry the loading conditions shown. Knowing that the allowable normal stress for the grade of steel to be used is 160 MPa, determine the minimum required section modulus for the beam. To get you started, I've given you the shear diagram.

(a) $3.25 \times 10^{-4}$ m$^3$
(b) $1.300 \times 10^{-4}$ m$^3$
(c) $4.23 \times 10^{-4}$ m$^3$
(d) $3.63 \times 10^{-4}$ m$^3$
(e) $4.64 \times 10^{-4}$ m$^3$

$$S_{min} = \frac{|M_{max}|}{\sigma_{allow}}$$

$$= \frac{67.6 \text{ kN} \cdot \text{m}}{160 \times 10^6 \text{ N/m}^2}$$

$$\boxed{S_{min} = 4.23 \times 10^{-4} \text{ m}^3}$$

Beam diagram labels: 20 kN, 50 kN, A, B, C, D, 3 m, 1 m, 1 m

Shear diagram labels: V, 52 kN, A, E, B, C, D, $x = 2.6$ m, $-8$ kN, $-58$ kN

$M_{max} @ E = $ area to left

$\underline{67.6 \text{ kN} \cdot \text{m}} = \frac{1}{2} (52)(2.6)$

Ron Kriz
ESM 2204, Fall 05
Test #3
(WORK OUT PROBLEM—40 points)

The unpressurized tank AB shown below has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Determine the maximum normal stress and the maximum shearing stress at point *a* on the top of the tank.

$2.2345 \times 10^{-4} m^4$

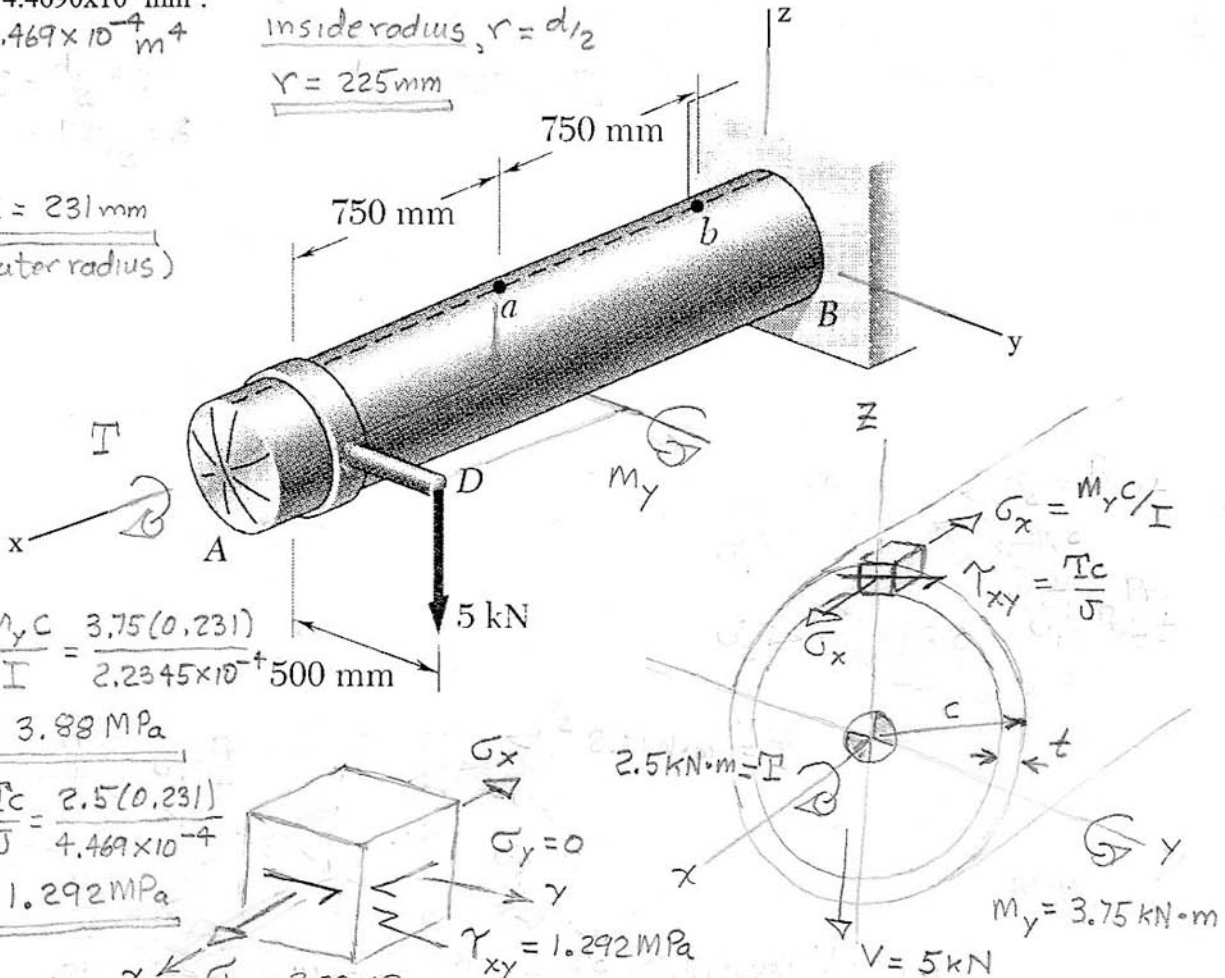For the tank, the area moment of inertia, $I$, is equal to $2.2345 \times 10^8$ mm$^4$. The polar moment of inertia, $J$, is equal to $4.4690 \times 10^8$ mm$^4$.

$4.469 \times 10^{-4} m^4$

Inside radius, $r = d/2$
$r = 225$ mm

$C = 231$ mm
(outer radius)

750 mm

750 mm

$z$

$b$

$a$

$B$

$y$

$T$

$M_y$

$z$

$D$

$x$

$A$

$5$ kN

$\sigma_x = \dfrac{M_y C}{I} = \dfrac{3.75(0.231)}{2.2345 \times 10^{-4}}$  500 mm

$\sigma_x = 3.88$ MPa

$\tau_{xy} = \dfrac{Tc}{J} = \dfrac{2.5(0.231)}{4.469 \times 10^{-4}}$

$\tau_{xy} = 1.292$ MPa

$\sigma_x$

$\sigma_y = 0$

$\gamma$

$\tau_{xy} = 1.292$ MPa

$\sigma_x = 3.88$ MPa

$\sigma_x = \dfrac{M_y C}{I}$

$\tau_{xy} = \dfrac{Tc}{J}$

$\sigma_x$

$c$

$t$

$2.5$ kN·m $= T$

$x$

$y$

$M_y = 3.75$ kN·m

$V = 5$ kN

$\sigma_y = 0$

$1.292$

$3.88$

$x$

$\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}$, $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\dfrac{3.88 - 0}{2}\right)^2 + 1.292^2}$

$\sigma_{avg} = \dfrac{3.88}{2}$

$R = 2.33$ MPa

$\sigma_{avg} = 1.94$ MPa

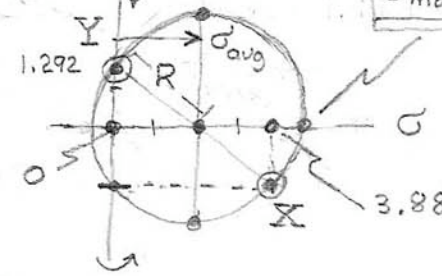$\sigma_{max} = 1.94 + 2.33$

$\boxed{\sigma_{max} = +4.27 \text{ MPa}}$

$\sigma_{max} = \sigma_{avg} + R$
$\sigma_{min} = \sigma_{avg} - R$
$\sigma_{min} = 1.94 - 2.33$
$\sigma_{min} = -0.39$ MPa

$Y$

$1.292$  $R$  $\sigma_{avg}$

$\sigma$

$X$

$3.88$

Because $\sigma_{max}^+$ and $\sigma_{min}^-$

$\tau_{max}^{in-plane} = \dfrac{1}{2}(\sigma_{max}^- \sigma_{min}^-) = R$

$\boxed{\tau_{max} = 2.33 \text{ MPa}}$

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