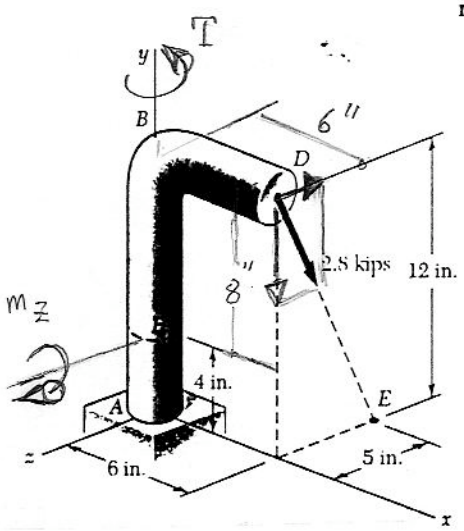
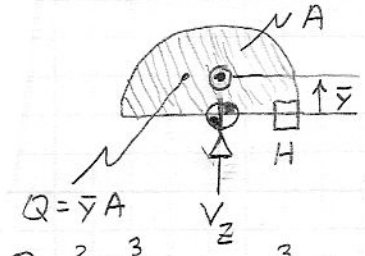
Problem 8.44

8.44 A 2.8-kip force is applied as shown to the 2.4-in.-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

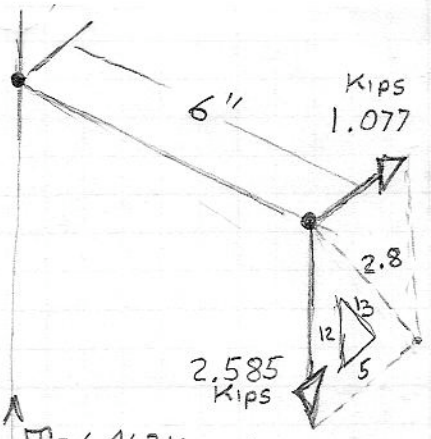


$$\frac{F_y}{12} = \frac{2.8}{13} = 2.585 \text{ kips}$$

$$\frac{F_x}{5} = \frac{2.8}{13} = 1.077 \text{ kips}$$



$$Q = \frac{2}{3}r^3 = 1.152 \text{ in}^3 = Q$$



$T = 6.462 \text{ kip·in}$

$M_z = 15.51 \text{ kip·in}$

$d = 2.4 \text{ in.}, \quad r = 1.2 \text{ in.}$

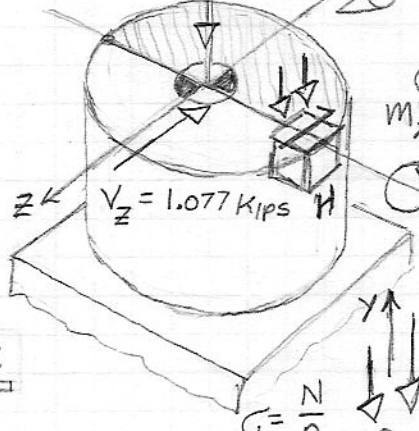
$A = \pi r^2 = 4.524 \text{ in}^2 = A$

$I = \frac{\pi}{4} r^4 = 1.629 \text{ in}^4 = I$

$J = 2I = 3.257 \text{ in}^4 = J$

$2.585 \text{ kips} = N$

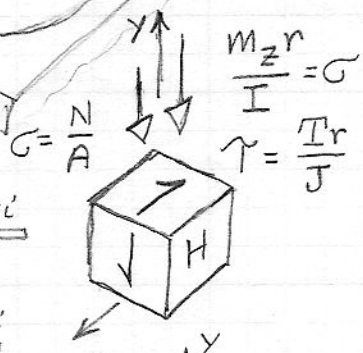
(NOT USED)
$M_x = 8.6152 \text{ kip·in}$



At point H:

$$\sigma = -\frac{N}{A} - \frac{M_z r}{I} = -\frac{2.585}{4.524} - \frac{15.51(1.2)}{1.629}$$

$$\sigma = -12.0 \text{ ksi}$$



$$\tau = +\frac{Tr}{J} + \frac{VQ}{I(d)} = \frac{6.462(1.2)}{3.257} + \frac{1.077(1.152)}{1.629(2.4)}, \quad \tau = 2.698 \text{ ksi}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = -6.0 \text{ ksi}, \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 6.677 \text{ ksi}$$

$\sigma_{max} = \sigma_{avg} + R$

$\sigma_{min} = \sigma_{avg} - R$

$\sigma_{max} = +0.677 \text{ ksi}$

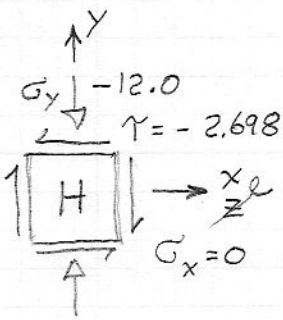
$\sigma_{min} = -12.677 \text{ ksi}$

because $\sigma_{max, min}$ (+, −).

$\tau_{max}^{\text{in-plane}} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

also $= R$

$\tau_{max} = 6.677 \text{ ksi}$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{|\sigma_x - \sigma_y|} = \frac{2(2.698)}{12.0} = 0.4497$$

$\theta_p = 12.1°$