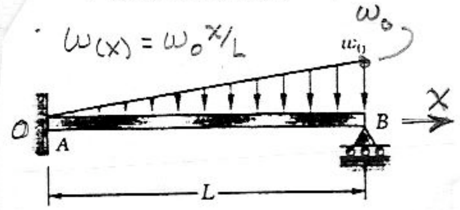
Problem 9.22

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



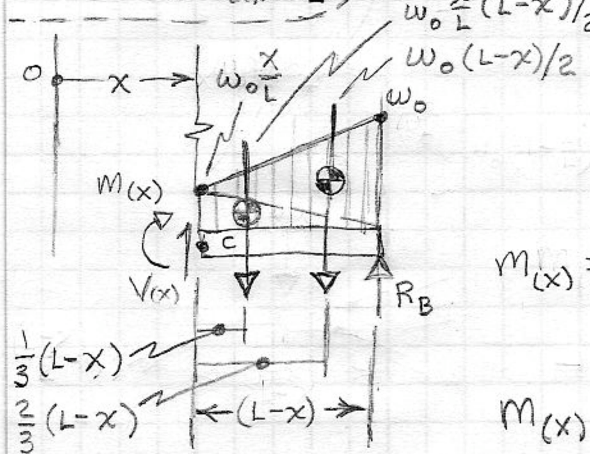
Reaction at B is statically indeterminate so label it R_B and proceed as usual

Determine, $M(x)$ function with R_B as an unknown

[1] [$x=0, y=0$]

[2] [$x=0, dy/dx=0$]

[3] [$x=L, y=0$]



$$\overset{+}{\circlearrowleft} \sum M_C^{@} = 0 = -M(x) + R_B(L-x)$$

$$+\, w_o (L-x)/2 \cdot \tfrac{2}{3}(L-x) \Big\}\ \text{upper triangle} \qquad \tfrac{2}{6}$$

$$+\, w_o \tfrac{x}{L}(L-x)/2 \cdot \tfrac{1}{3}(L-x) \Big\}\ \text{lower triangle} \qquad \tfrac{1}{6}$$

$$M_{(x)} = R_B(L-x) - \frac{w_o}{6L}\underbrace{\left[2L(L-x)^2 + x(L-x)^2\right]}_{\text{Simplify}}$$

$$\underline{M(x) = R_B(L-x) - \frac{w_o}{6L}\left[x^3 - 3L^2x + 2L^3\right]} = EI\frac{d^2y_{(x)}}{dx^2}$$

Integrate

$$EI\, dy/dx = R_B\left(Lx - x^2/2\right) - \frac{w_o}{6L}\left(x^4/4 - \tfrac{3}{2}L^2x^2 + 2L^3x\right) + C_1$$

Integrate

$$EI\, Y(x) = R_B\left(Lx^2/2 - x^3/6\right) - \frac{w_o}{6L}\left(x^5/20 - \frac{L^2x^3}{2} + L^3x^2\right) + C_1x + C_2$$

use BC [1]: $x=0,\ y=0 \quad \therefore\ \underline{C_2 = 0}$

use BC [2]: $x=0,\ dy/dx = 0 \quad \therefore\ \underline{C_1 = 0}$

use BC [3]: $x=L,\ y=0$: $0 = R_B L^3\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{w_o L^4}{6}\left(\frac{1}{20} - \frac{1}{2} + 1\right)$

$$\boxed{R_B = \frac{11}{40} w_o L}$$