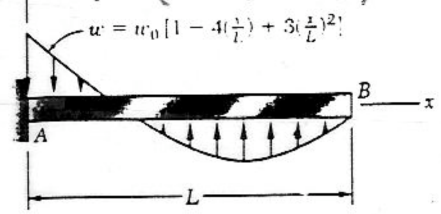
Problem 9.18

9.18 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the free end.

$y \quad \omega_{(x)} = \omega_0\left(1 - 4\frac{x}{L} + 3\frac{x^2}{L^2}\right)$



[1] [$x=0, y=0$]
[2] [$x=0, dy/dx=0$]
[3] [$x=L, V=0$]
[4] [$x=L, M=0$]

$$\frac{dV_{(x)}}{dx} = -\omega_{(x)} = -\omega_0\left(1 - 4\frac{x}{L} + 3\frac{x^2}{L^2}\right)$$

$$V_{(x)} = -\omega_0\left(x - 2\frac{x^2}{L} + \frac{x^3}{L^2}\right) + C_V$$

Use BC [3] @ $x = L$, $V = 0$

$$0 = -\omega_0\left(L - 2L + L\right) + C_V \;;\; \therefore \boxed{C_V = 0}$$

$$\frac{dM_{(x)}}{dx} = V_{(x)} = -\omega_0\left(x - 2\frac{x^2}{L} + \frac{x^3}{L^2}\right)$$

$$M_{(x)} = -\omega_0\left[\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right] + C_M = 0$$

Use BC [4] $x = L$, $M = 0$

$$0 = -\omega_0\left[\tfrac{1}{2}L^2 - \tfrac{2}{3}L^2 + \tfrac{1}{4}L^2\right] + C_M$$

$$\therefore \boxed{C_M = \frac{\omega_0 L^2}{12}}$$

$$EI\frac{d^2y_{(x)}}{dx^2} = \boxed{M_{(x)} = -\omega_0\left(\frac{x^2}{2} - \frac{2}{3}\frac{x^3}{L} + \frac{x^4}{4L^2} - \frac{L^2}{12}\right)}$$

Integrate $\quad EI\,dy_{(x)}/dx = -\omega_0\left(\frac{x^3}{6} - \frac{x^4}{6L} + \frac{x^5}{20L^2} - \frac{L^2 x}{12}\right) + C_1 = 0$

use BC [2], $x = 0$ $\quad dy/dx = 0$ $\quad \therefore \boxed{C_1 = 0}$

Slope: $\boxed{\dfrac{dy_{(x)}}{dx} = -\dfrac{\omega_0}{EI}\left(\dfrac{x^3}{6} - \dfrac{x^4}{6L} + \dfrac{x^5}{20L^2} - \dfrac{L^2 x}{12}\right)} \quad (1)$

Integrate $\quad EI\,y_{(x)} = -\omega_0\left(\frac{x^4}{24} - \frac{x^5}{30L} + \frac{x^6}{120L^2} - \frac{L^2x^2}{24}\right) + C_2 = 0$

use BC [1], $x = 0$, $y = 0$ $\quad \therefore \boxed{C_2 = 0}$

Elastic curve: $\boxed{y_{(x)} = -\dfrac{\omega_0}{EIL^2}\left(\dfrac{L^2}{24}x^4 - \dfrac{L}{30}x^5 + \dfrac{1}{120}x^6 - \dfrac{L^4}{24}x^2\right)}$

Deflection at B $x = L$ $\quad y^B_{(x=L)} = -\dfrac{\omega_0}{EIL^2}\left[\dfrac{1}{24}L^6 - \dfrac{1}{30}L^6 + \dfrac{1}{120}L^6 - \dfrac{1}{24}L^6\right]$ $\quad \boxed{y^B_{x=L} = -\dfrac{\omega_0 L^4}{40EI}}$