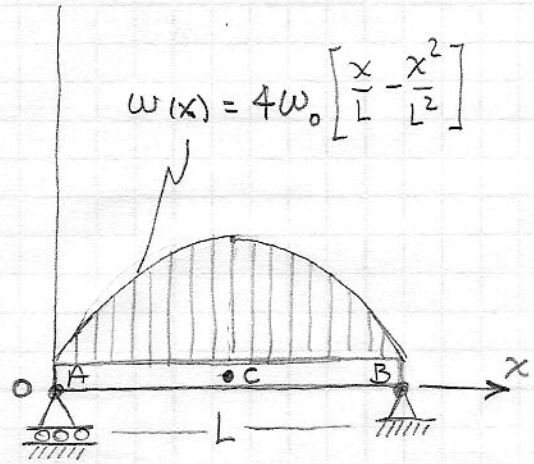
Problem 9.17

9.17 For the beam and loading shown determine (a) the equation of the elastic curve, (b) slope at A, (c) deflection at mid-span.



[1] [x=0, M=0] [3] [x=L, M=0]

[2] [x=0, y=0] [4] [x=L, y=0]

Determine, $M(x)$

$$\frac{dV(x)}{dx} = -w(x) = \frac{w_0}{L}(4x^2 - 4Lx)$$

$$V(x) = \frac{w_0}{L}\left(\frac{4}{3}x^3 - 4L\frac{x^2}{2}\right) + C_1$$

$$\frac{dM(x)}{dx} = V(x) = \frac{w_0}{L}\left(\frac{4}{3}x^3 - 2Lx^2\right) + C_1$$

$$M(x) = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3\right) + C_1x + C_2 = 0$$

Use BC [1] $x=0$, $M=0$ $\therefore C_2 = 0$

Use BC [3] $x=L$, $M=0$

$$0 = \frac{w_0}{L^2}\left(L^4/3 - 2/3\,L^4\right) + C_1L \quad \therefore C_1 = \frac{w_0L^2}{3}$$

$$M(x) = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x\right) \quad (1)$$

Determine elastic curve $EI\,d^2y/dx^2 = M(x) = \frac{w_0}{L^2}\left(x^4/3 - \frac{2Lx^3}{3} + \frac{L^3x}{3}\right)$

integrate $EI\,dy/dx = \frac{w_0}{L^2}\left(x^5/15 - \frac{Lx^4}{6} + \frac{L^3x^2}{6}\right) + C_3$

integrate again

$$EI\,y(x) = \frac{w_0}{L^2}\left(x^6/90 - Lx^5/30 + L^3x^3/18\right) + C_3x + C_4$$

Use BC [2] @ $x=0$, $y=0$ $= \frac{w_0}{L^2}(0 - 0 + 0) + C_3 0 + C_4$ $\therefore C_4 = 0$

Use BC [4] @ $x=L$, $y=0$ $= \frac{w_0}{L^2}\left(L^6/90 - L^6/30 - L^6/18\right) + C_3L$ $\therefore C_3 = -\frac{w_0L^3}{30}$

elastic curve: $\boxed{y(x) = \frac{w_0}{EIL^2}\left(x^6/90 - Lx^5/30 + L^3x^3/18 - L^5x/30\right)}$ (1) ← subst. back

Calc. slope

$$dy(x)/dx = \frac{w_0}{EIL^2}\left(x^5/15 - Lx^4/6 + L^3x^2/6 - L^5/30\right) \quad (2)$$

Slope at A, $x=0$, eqn (2) reduces to, $\boxed{dy/dx\,\big|^A_{x=0} = \frac{w_0L^3}{30EI}}$: Slope

Deflection at midspan, $x=L/2$

$$y_{(x=L/2)} = \frac{w_0L^4}{EIL^2}\left\{\frac{1}{90}\left(\frac{1}{2}\right)^6 - \frac{1}{30}\left(\frac{1}{2}\right)^5 + \frac{1}{18}\left(\frac{1}{2}\right)^3 - \frac{1}{30}\left(\frac{1}{2}\right)\right\}$$

$$\boxed{y^C_{x=L/2} = \frac{61}{5760}\frac{w_0L^4}{EI}}$$