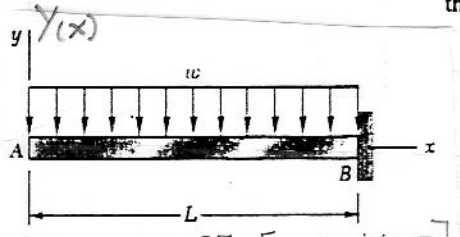
Problem 9.4

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

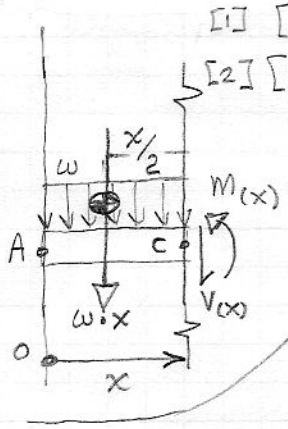


Determine: $M_{(x)}$ Function for integration

$$\left(+\circlearrowleft\right)\sum M_{@C} = 0 = \frac{x}{2}\cdot wx + M_{(x)}, \quad M_{(x)} = -\frac{wx^2}{2}$$

[1] $[x=L,\ Y=0]$

[2] $[x=L,\ dy/dx=0]$



$$EI\,\frac{d^2y_{(x)}}{dx^2} = M_{(x)} = -\frac{wx^2}{2}$$

integrate

$$EI\,\frac{dy_{(x)}}{dx} = -\frac{1}{2}\frac{1}{3}wx^3 + C_1 = 0 \qquad (x \to L^3)$$

Use BC [2] @ $x=L$, $dy/dx = 0$ → $C_1 = \frac{wL^3}{6}$ (Subst. back)

$$EI\,dy_{(x)}/dx = -\frac{wx^3}{6EI} + \frac{wL^3}{6} \quad (1)\ \text{slope}$$

integrate

$$EI\,y_{(x)} = -\frac{w}{24EI}x^4 + \frac{wL^3}{6}x + C_2 = 0 \qquad (x \to L^4,\ x \to L)$$

Use BC [1]: @ $x=L$, $Y=0$ → $C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4$

$$C_2 = -\frac{3}{24}wL^4$$

Substitute back

$$\boxed{y_{(x)} = -\frac{w}{24EI}\left(x^4 - 4L^3x + 3L^4\right)} \quad (2)$$

elastic curve "deformation"

Deformation at "Free-end", A $(x=0)$

$$y_{(x=0)} = -\frac{w}{24EI}\left(0 - 0 + 3L^4\right)$$

$$\boxed{y^A_{x=0} = -\frac{wL^4}{8EI}}$$

Slope at "Free-end", A $(x=0)$

$$EI\left.\frac{dy_{(x)}}{dx}\right|_{x=0} = -\frac{w\cdot 0}{6EI} + \frac{wL^3}{6}$$

$$\boxed{\left.\frac{dy_{(x=0)}}{dx}\right|_{x=0} = \frac{wL^3}{6EI}}$$