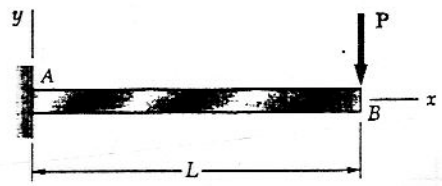
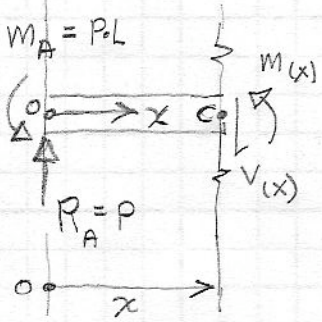
Problem 9.1

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



Determine: $M_{(x)}$ Function For integration

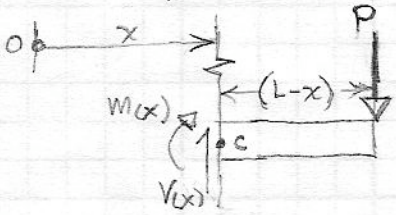


$\circlearrowleft + \sum M_C @ = 0 = +PL - Px + M_{(x)}$, $M_{(x)} = -P(L-x)$

[OR]

$= 0 = -M_{(x)} - (L-x)P$, $M_{(x)} = -P(L-x)$

[OR]



$$EI\frac{d^2y_{(x)}}{dx^2} = M_{(x)} = -PL + Px$$

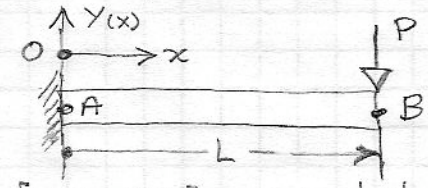
integrate

$$EI\frac{dy_{(x)}}{dx}_{(\text{slope})} = -PLx + \frac{1}{2}Px^2 + C_1 = 0$$

Use BC [2] @ $x=0$, $\frac{dy_{(x)}}{dx}_{(\text{slope})} = 0$ $\therefore C_1 = 0$

$$EI = \frac{dy_{(x)}}{dx} = -PLx + \frac{1}{2}Px^2 \quad (1)\ \text{slope}$$

Boundary Conditions (BCs)



[x=0, y=0] [1]

[x=0, $dy/dx = 0$] [2]

none x=L

integrate

$$EI\,Y_{(x)} = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_2 = 0$$

displacement

use BC [1] @ $x=0$, $Y_{(x)} = 0$ $\therefore C_2 = 0$

$$\boxed{Y_{(x)} = \frac{Px^2}{6EI}(3L-x)}$$ (2) "deformation" elastic curve

Displacement @ x=L

Substitute

$$\boxed{Y^B_{(x=L)} = -PL^3/(3EE)}$$ deflection at B

Slope @ x=L $\left.\frac{dy_{(x)}}{dx}\right|_{x=L}^{B} = -PL(L) + \frac{1}{2}P(L)^2$,

$$\boxed{\left.\frac{dy}{dx}\right|_{x=L}^{B} = -\frac{PL^2}{2EI}}$$

slope at B