ESM 2204 - FINAL EXAMINATION

MECHANICS OF DEFORMABLE BODIES

EXAMPLE

NAME
(print) ____
last first initial

PLEDGE (signature): On my honor I have neither given nor received unauthorized aid on this test.

____

INSTRUCTIONS:

Closed book, closed notes.

There are 13 questions on this exam - check for completeness. Part I (40%) consists of 10 short-answer problems (each problem is worth equal credit). Part II (60%) consists of three work-out problems (each problem is worth equal credit).

Turn in your results in the following order:

Exam questions (signed)

Ten short-answer problems (in order)

Three work-out problems (in order)

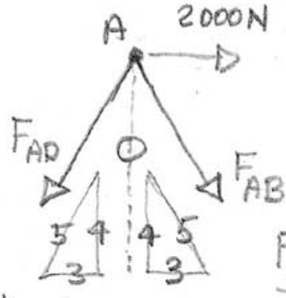
Staple the above in the upper left-hand corner.

CHECK FOR COMPLETENESS

Practice Final Exam #1, pfe1.pdf

Part I

1. Determine the change in the length of member AB of the truss shown. Neglect the effects of buckling. Member AB has a solid circular cross section with a 25 mm diameter; $E = 200$ GPa.

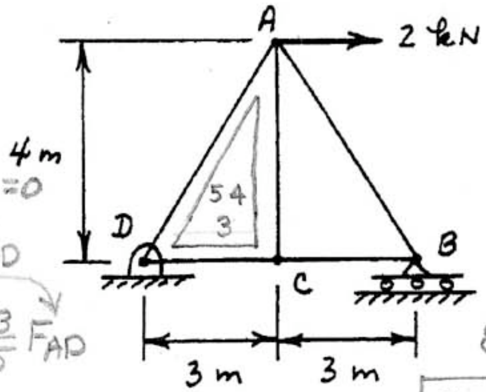


$\Sigma F_y = 0$

$F_{AB} = -F_{AD}$

$\overset{+}{\rightarrow} \Sigma F_y = 0 = +2000 + \frac{3}{5} F_{AB} - \frac{3}{5} F_{AD}$

$F_{AB} = -10{,}000/6$



$A = \pi r^2 = \pi (0.0125\text{m})^2$

$A = 0.4909 \times 10^{-3}\ \text{m}^2$

$L_{AB} = 5\text{m}$

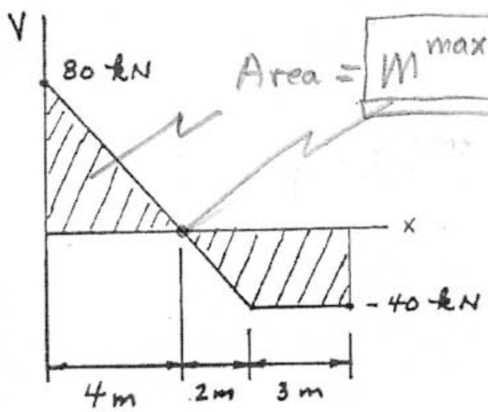
$E = 200 \times 10^9\ \text{N/m}^2$

$\delta_{AB} = F_{AB} L_{AB} / AE$

$\delta_{AB} = -\dfrac{10{,}000/6 \cdot 5}{(0.4909)(200)\,10^6}$

$\boxed{\delta_{AB} = -84.9 \times 10^{-6}\ \text{m}}$

2. The shear diagram for a beam simply supported at the ends is shown. There are no concentrated moments acting on the beam. Determine the magnitude of the maximum bending moment.



Area = $\boxed{M^{max} = 160\ \text{KN-m}}$

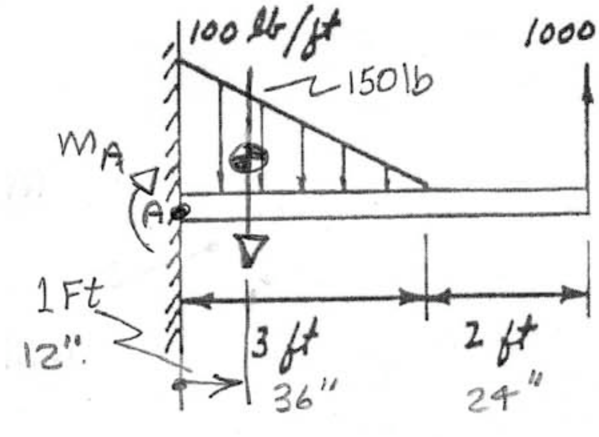
3. For the beam loaded as shown, determine the moment at the wall.

$$\circlearrowleft^{+} \sum M_{@A} = 0 =$$

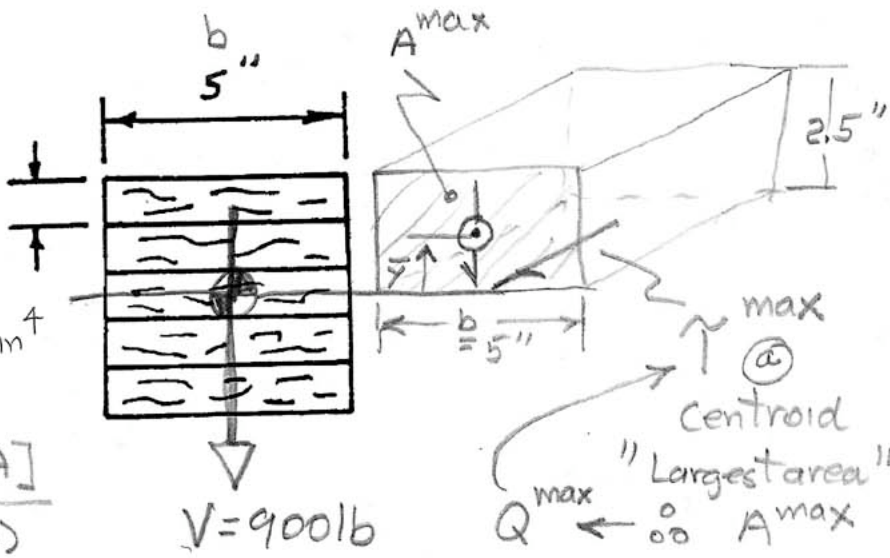
$$-M_A - (12'')\,150\text{ lb} + (60'')\,1000\text{ lb} = 0$$

$$-1800 \qquad +60{,}000$$

$$\boxed{M_A = +58{,}200\text{ in-lb}}$$



4. A beam is made by gluing five 1 × 5 in. boards together. For a vertical shear of 900 lb., determine the maximum shearing stress in the wood.



$$I = \frac{bh^3}{12} = \frac{5(5)^3}{12} = 52.08\text{ in}^4$$

$$\tau = \frac{VQ^{max}}{It} = \frac{V[\bar{y}\cdot A]}{I\,(b)}$$

$$\tau^{max} = \frac{900\text{ lb}\,[(1.25)(2.5)\cancel{(5.0)}]}{52.08\,\cancel{(5)}} \Rightarrow \boxed{\tau^{max} = 54\text{ lb/in}^2}$$

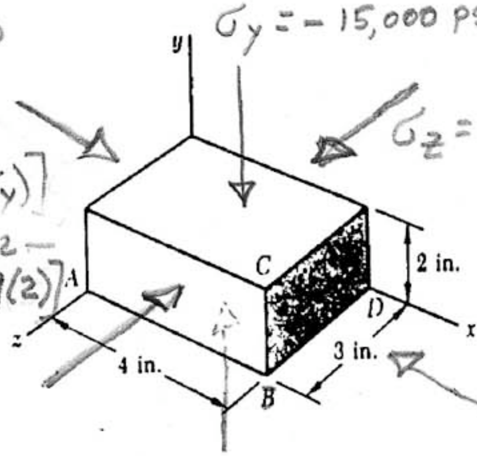
5. The steel block ($E = 29 \times 10^6$ psi, $v = 0.29$) is subjected to a uniform pressure of 15 ksi on all of its faces. Determine the change in length of the 3 in. side of the block.

$$\epsilon_z = \frac{\Delta \overline{BD}}{\overline{BD}} = \frac{\Delta \overline{BD}}{3''}$$

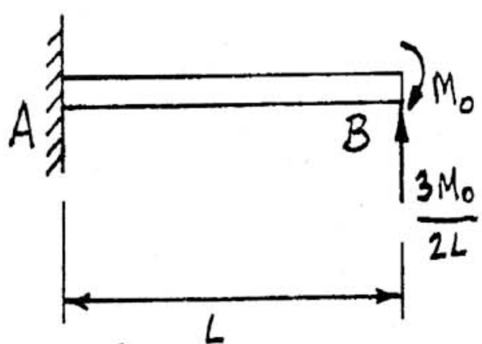
$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\frac{\Delta \overline{BD}}{3} = \frac{-15 \times 10^{3}}{29 \times 10^{6}}\left[1 - \underbrace{0.29(2)}_{+0.42}\right]$$

$$\boxed{\Delta \overline{BD} = 0.65 \times 10^{-3} \text{ in.}}$$



6. Determine the slope at the end B of the beam loaded as shown.



Case 1 $\quad \theta_B = -\frac{PL^2}{2EI} = +\frac{(3M_o/2L)\,L^2}{2EI}$

Case 3 $\quad \theta_B = -\frac{ML}{EI} = -\frac{M_oL}{EI}$

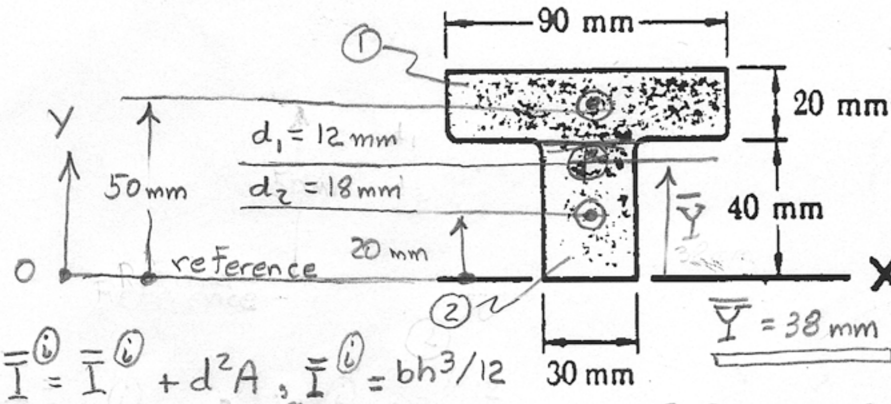
Superpose

+

$$\theta_B = \frac{3}{4}\frac{M_oL}{EI} - \frac{M_oL}{EI} \Rightarrow \boxed{\theta_B = -\frac{M_oL}{4EI}}$$

7. Determine the moment of inertia of the shaded area with respect ot the x-axis.

Region	$\bar{y}$ (mm)	A (mm^2)	$\bar{y}A$ (mm^3)	d^2 (mm^2)	Ad^2 ($\times 10^3$ mm^4)
1	50	1800	90,000	144	259.2
2	20	1200	24,000	324	388.8
Σ	–	3000	114,000	–	648



$$\bar{Y} = 38 \text{ mm}$$

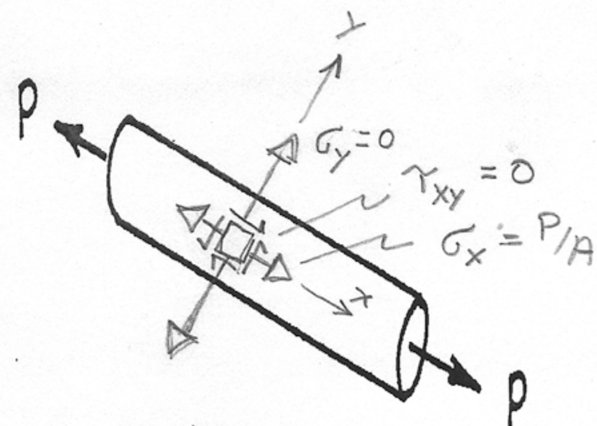
$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{114}{3}$$

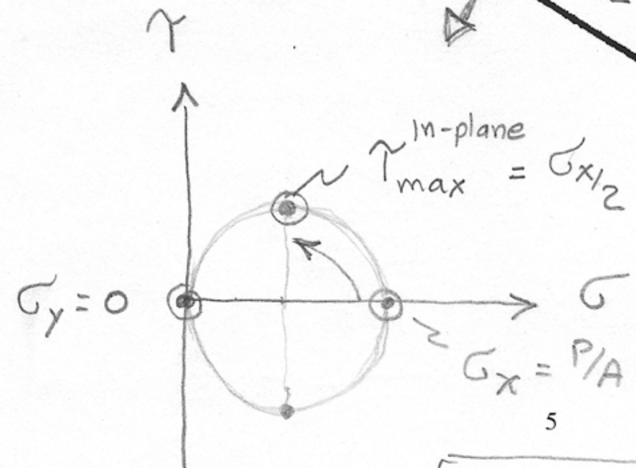
$$\boxed{\bar{I}_x = 868 \times 10^3 \text{ mm}^4}$$

$$\bar{I}_x^{(i)} = \bar{I}^{(i)} + d^2 A, \quad \bar{I}^{(i)} = bh^3/12$$

$$\bar{I}_x = \Sigma \bar{I}^{(i)} = \Big[\underbrace{90(20)^3/12}_{60\times10^3} + 259.2\times10^3\Big]^{(1)} + \Big[\underbrace{30(40)^3/12}_{160\times10^3} + 388.8\times10^3\Big]^{(2)}$$

8. The 2-in. diameter solid circular shaft is subjected to a centric force P = 10,000 lb. Determine the maximum shearing stress.





$$\sigma_x = P/A = \frac{10{,}000}{\pi(1)^2}$$

$$\boxed{\tau_{max} = \frac{5{,}000}{\pi} \text{ psi}}$$

9. The column has a 2 × 2-in. cross section and is made of aluminum ($E = 10 \times 10^6$ psi, $\alpha = 12 \times 10^{-6}/°F$). Determine the change in the temperature that will cause the column to buckle.

$$\frac{\pi^2 \cancel{E} I 4}{A \cancel{E} L^2} = \alpha \Delta T$$

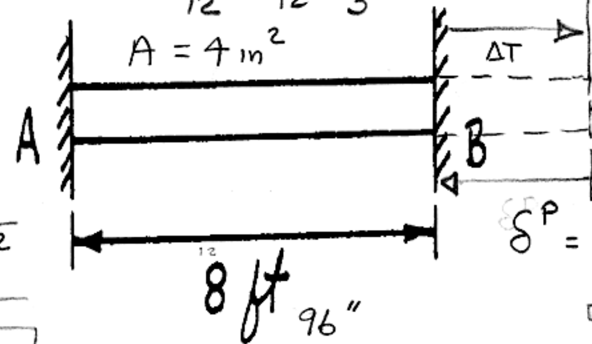
$$I = \frac{bh^3}{12} = \frac{16}{12} = \frac{4}{3}\,\text{in}^4$$

$$A = 4\,\text{in}^2$$

$$\delta^T = \alpha \cdot \Delta T \cdot L$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e = L/2$$



$$\Delta T = \frac{I \pi^2 4}{A \alpha L^2}$$

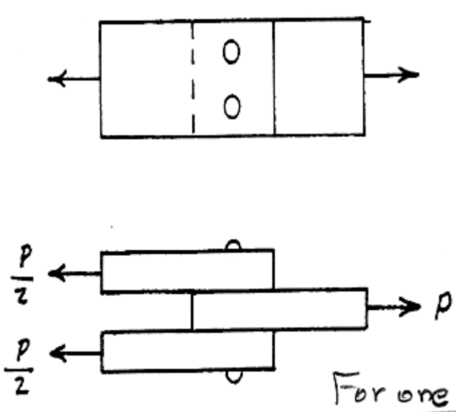
$$\delta^P = \frac{P_{cr} \cancel{L}}{AE} = \alpha \Delta T \cancel{L}$$

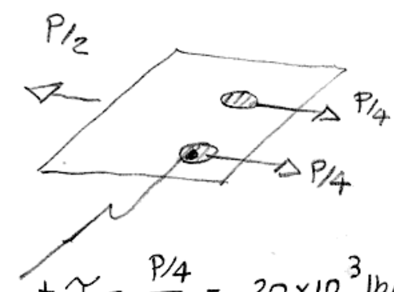
$$\Delta T = \frac{\cancel{4}}{3} \cdot \frac{\pi^2 \cdot 4}{\cancel{4} \cdot 12 \times 10^{-6} \cdot (96)^2}$$

$$331.776 \times 10^{-3}$$

$$\boxed{\Delta T = 119°F}$$

10. Two 0.5-in. diameter rivets are used to connect the three steel plates. If the allowable shearing stress in the rivets is 20 ksi, determine the largest load P that may be applied.





For one rivet: $\tau = \frac{P/4}{A} = 20 \times 10^3\ \text{lb/in}^2$

$$A = \pi (0.25)^2 = \cancel{16\pi}\quad \pi/16 \text{ oops}$$

$$\cancel{\frac{P}{64\pi} = 20 \times 10^6}$$

$$\cancel{P = 64(\pi)(20 \times 10^6)}$$

$$\cancel{\boxed{P = 4.02 \times 10^9\ \text{lb}}}$$

Correction

6

$$A = \pi/16$$

$$\frac{P/4}{\pi/16} = 20 \times 10^3$$

$$\frac{16}{4} P = \pi \cdot 20 \times 10^3$$

$$4P = \pi \cdot 20 \times 10^3$$

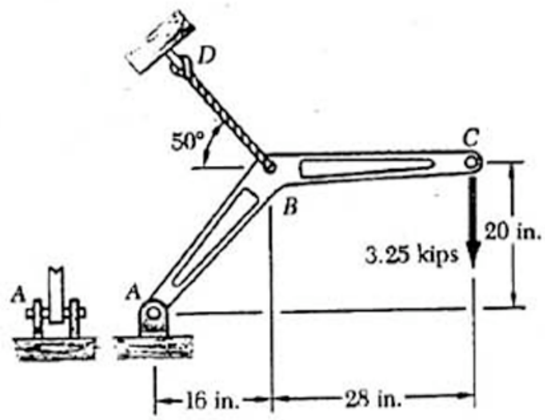
$$P = 5 \cdot \pi \times 10^3 = 15.71 \times 10^3\ \text{lb}$$

Part II

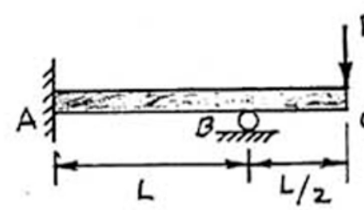
1. Cable BD has a breaking strength of 20 kips, and the pin at A has a diameter of 0.375 in. and is made of steel with an ultimate shearing stress of 50 ksi. Determine the factor of safety for the loading shown.

2. The compressed air vessel shown in the figure for Problem 2 has an outside diameter of 462 mm and a wall thickness of 6 mm. Knowing that the gage pressure inside the vessel is 120 kPa, determine the magnitude of the maximum normal stress at point K.

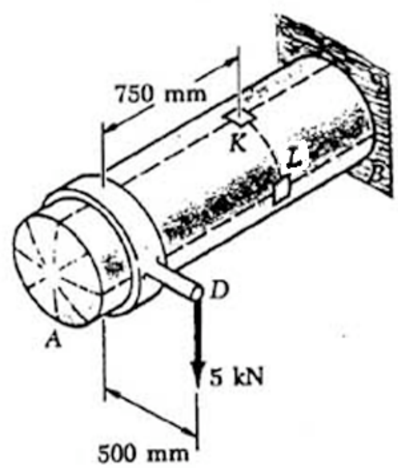
3. Determine the reactions at A and B for the beam loaded and supported as shown in the figure for Problem 3.



PROBLEM 1

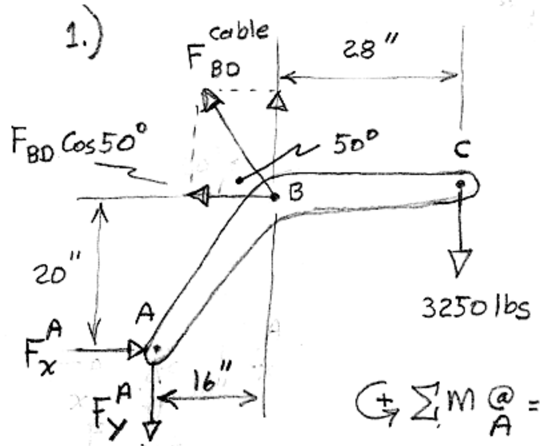


PROBLEM 3



PROBLEM 2

1.)



$F_{BD}^{\text{Cable Ultimate}} = 20{,}000 \text{ lbs}$

Pin A, dia = 0.375 in

Pin A, area = $\pi (0.1875)^2 = 0.110 \text{ in}^2$

$\tau_A^{ult} = 50{,}000 \text{ lb/in}^2$

$\circlearrowleft^{+} \sum M_{@A} = 0 = +20(F_{BD} \cos 50°) + 16(F_{BD} \sin 50°) - 44(3250)$

$+ 12.86\, F_{BD} + \underset{12.86}{12.26}\, F_{BD} = 143{,}000$

$25.12\, F_{BD} = \text{"}$

$\underline{F_{BD}^{cable} = 5693 \text{ lbs}}$

$F.S. = \dfrac{F_{BD}^{ult}}{F_{BD}} = \dfrac{20{,}000}{5{,}693}$

$\boxed{F.S.^{cable} = 3.51}$

$\overset{+}{\rightarrow} \sum F_x = 0 = +F_x^A - F_{BD} \cos 50°$

$\underline{F_x^A = 3659 \text{ lb}}$

$+\uparrow \sum F_y = 0 = -F_y^A + \underbrace{F_{BD} \sin 50°}_{4361} - 3250$

$\underline{F_y^A = 1111 \text{ lb}}$

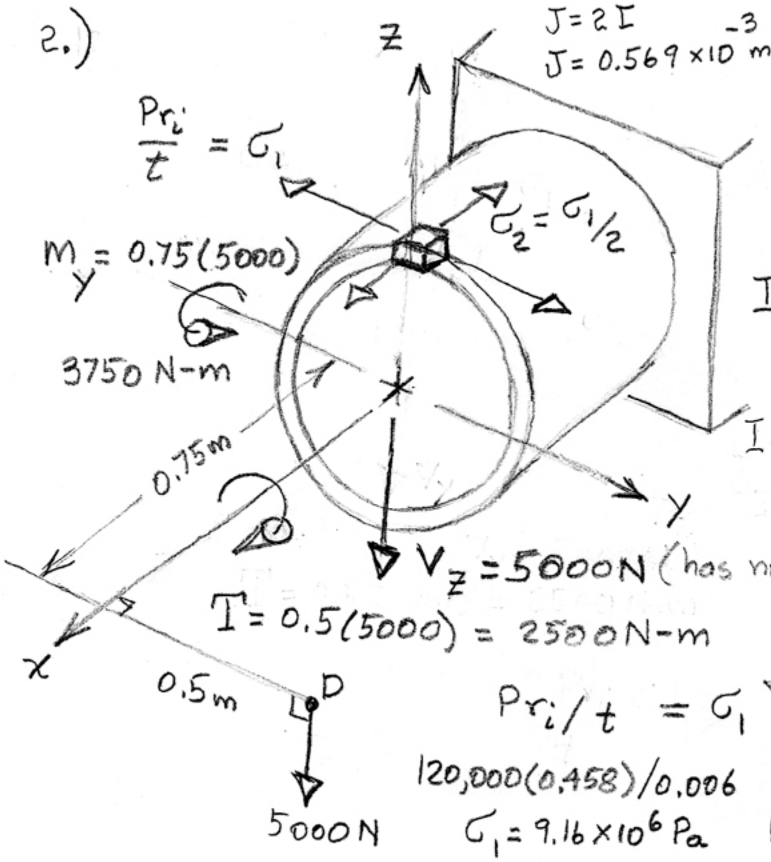
$F^A = \sqrt{(F_x^A)^2 + (F_y^A)^2} = \underline{3824 \text{ lb} = F^A}$

$13.39 + 1.234$

$\tau^A = \dfrac{F^A/2}{A_{pin}} = \dfrac{3824/2}{0.110} = 17{,}382 \text{ psi}$

$F.S.^{Pin} = \tau_A^{ult} / \tau A = \dfrac{50{,}000}{17{,}382} \Rightarrow \boxed{F.S.^{Pin} = 2.88}$

2.)



$J = 2I$

$J = 0.569 \times 10^{-3}\ m^4$

	Dia.	radius
$r_o =$	~~462~~ mm	231 mm $= r_o$
$t =$	6 mm	-6
		225 mm $= r_i$
$r_i =$	~~458~~ mm	

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}\left((0.231)^4 - (0.225)^4\right)\ [m]$$

$$= \frac{\pi}{4}(2.8474 - 2.5629)\times 10^{-3}$$

$$I = 0.285 \times 10^{-3}\ m^4$$

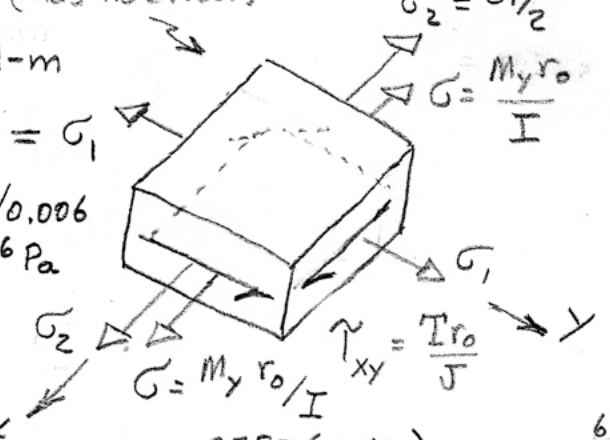
$$T = 0.5(5000) = 2500\ N\text{-}m$$

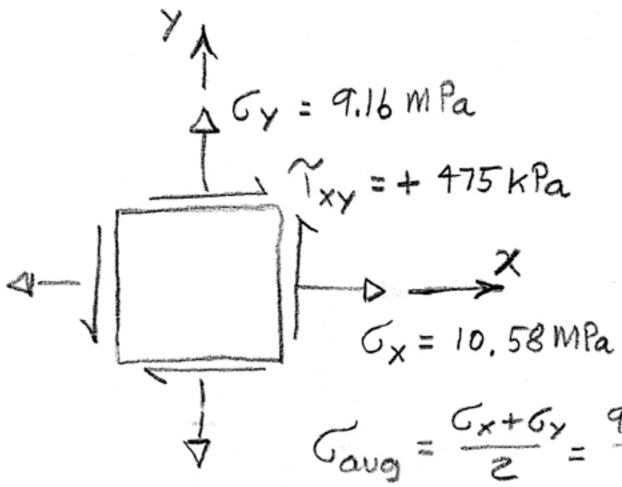
$$Pr_i/t = \sigma_1$$

$$120{,}000(0.458)/0.006$$

$$\sigma_1 = 9.16 \times 10^6\ Pa$$

$\sigma_2 = 4.58 \times 10^6\ Pa$





$$\sigma = \frac{3750(0.462)}{1.217\times 10^{-3}} = 1.42 \times 10^6\ Pa$$

$$\tau_{xy} = \frac{T r_o}{J} = \frac{2500(0.462)}{0.569 \times 10^{-3}}$$

$$\underline{\tau_{xy} = 2.030\ MPa}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{9.16 + 10.58}{2}$$

$$\underline{\sigma_{avg} = 9.87\ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{9.16 - 10.58}{2}\right)^2 + 2.030^2}$$

$$\sqrt{0.504 + 4.1204}$$

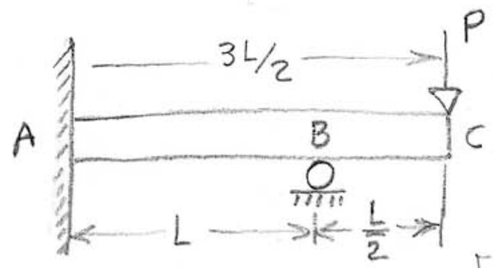
$$4.624$$

$$\underline{R = 2.1504\ MPa}$$

$$\sigma_{max} = \sigma_{avg} + R$$

$$\boxed{\sigma_{max} = 12.02\ MPa}$$

3.)



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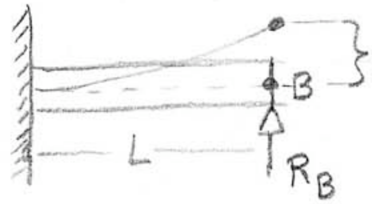
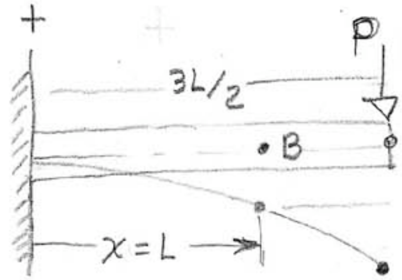
$\left[\text{Case 1}: \; \delta = PL^3/3E \right]$



$$\delta_B^{R_B} = + \frac{R_B L^3}{3EI}$$

+

$\left[\text{Case 1}: \; Y_{(x)} = \frac{P}{6EI}\left(x^3 - 3Lx^2\right) \right]$



$$Y^B_{(x=L)} = + \frac{P}{6EI}\left(L^3 - 3\left(\tfrac{3L}{2}\right)L^2\right)$$

$$\delta_B = 0 = \delta_B^{R_B} + Y^B_{(x=L)} = \frac{R_B L^3}{3\cancel{EI}} + \frac{P}{6\cancel{EI}}\left(L^3 - 3\left(\tfrac{3L}{2}\right)L^2\right) = 0$$

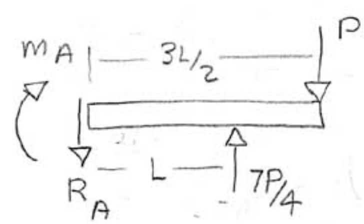
$$0 = \frac{R_B L^3}{3} + \frac{P}{6}\left(L^3 - \frac{9}{2}L^3\right); \quad R_B = -\frac{3}{6}P\left(1 - \frac{9}{2}\right)$$

$$R_B = -\frac{P}{2}\left(\frac{2}{2} - \frac{9}{2}\right)$$

$$\boxed{R_B = +\frac{7}{4}P}$$

$$\boxed{R_A = \frac{3}{4}P \downarrow}$$

Overall FBD



$$\circlearrowleft + \sum M_{@A} = 0 = -M_A + \frac{7PL}{4} - \frac{\cancel{3}^{6}PL}{\cancel{2}_{4}}$$

$$\boxed{\circlearrowleft M_A = PL/4}$$